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Short-Term Forecasting of GDP under Structural Changes

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Abstract

This paper proposes several models with time-varying parameters, estimated by Bayesian techniques used for the short-term forecasting of Croatian GDP. In addition to domestic variables, the models include EU GDP, so that the specificities of a small open economy have been taken into account. The predictive ability of the models is compared with the naive benchmark forecast. The results indicate that the modelling of time-varying parameters improves GDP forecasts in comparison with the naive benchmark model, and in addition, it has been established that mean forecast errors for all tested models with time-varying parameters are smaller than the errors of equally specified fixed parameter models.

JEL:

C32, E37, E47

Keywords:

GDP forecasts, Bayesian models with time-varying parameters

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Contents

1 Introduction

Over the past ten years or so, the Croatian economy has undergone extensive structural changes and changes in the trends of the main macroeconomic variables. For instance, the average annual growth rate of real gross domestic product in the period from 2000 to 2008 amounted to 4.5%, and because of several years of negative growth rates, this rate declined eventually to only 1.8%. As a result of the continuous fall in actual GDP, a fall even in the level of potential GDP was observed (Bokan and Ravnik, 2012). Also, gradual changes can be observed in the transmission of EU economic activity to domestic economic activity. In the pre-crisis period and during the first wave of the crisis (until the end of 2009) a considerable co-movement of business cycles and the current transmission of foreign economic activity to domestic[1] economic activity were observed. In the most recent period, however, such a transmission has been less visible. For instance, the brief exit from the recession observed in most European countries in 2010 was not seen in Croatia, while the recovery in 2011 was almost negligible. Changes are also obvious in international financial markets with an emphasis on the transmission of activities from the mentioned markets to the domestic financial and real sector. Kunovac (2013) shows that, apart from domestic fundamentals, the spillover of the fiscal crisis and contagion in the eurozone are becoming more important for the determination of the sovereign risk premium of the Republic of Croatia in 2011 and 2012. Such an exogenously triggered increase in the price of borrowing certainly had an impact on the budget and on fiscal policy and, finally, on domestic GDP. For this reason, the general VAT rate in Croatia was raised on two occasions after the onset of the recession, and the strong growth in budgetary expenditure from the period 2000 – 2011 slowed down in the past two years. Regarding credit activity, a period of accelerated growth in lending to the private sector and a significant foreign capital inflow can be identified in the period from 2002 to 2008, followed by the credit crunch (Krznar et al, 2011). For this reason, the CNB eased its monetary policy considerably, for instance, by the multiple lowering of the general reserve requirement with the parallel very expansive monetary policy of the European Central Bank.

Under the conditions of these significant structural changes and changes in economic policies, it is much more difficult to forecast economic activity than in the pre-crisis period. The future state of the economy is of crucial importance for the monetary policy-maker to be able to respond to future developments adequately and on time, using the available instruments. It is for this reason that this paper explores the possibility of the short-term forecasting of GDP by means of econometric time-varying parameter models (TVP), which provide the possibility of explicitly modelling the mentioned structural changes. The models will be estimated and forecasted iteratively on a moving sample that covers a five-year period, including high pre-crisis growth rates, the financial crisis and the sharp fall of GDP, as well as the so-called fiscal crisis. The performances of such models in terms of the mean forecast error will be compared with similar standard fixed parameter models.

1 For instance, Krznar and Kunovac (2010) show how a one percentage point increase in foreign (European) GDP shock causes an equivalent increase in domestic GDP during the first quarter, and a two percentage point increase after two years.

Bearing in mind the importance of the modelling of the mentioned structural changes, the paper aims to determine if there is any contribution in the use of the time-varying parameter models when modelling and forecasting GDP. The Bayesian estimation of time-varying parameters will be used for this purpose, as, for example, in Canova (2002), D'Agostino et al. (2010), Eickmeier et al. (2011) and Mumtaz et al. (2012). Forecasts for the US, the European Union and Great Britain were evaluated in all of the mentioned papers, while this paper aims to explore the predictive ability of such models for a much smaller post-transition economy such as the Croatian, which is strongly affected by external shocks. For this reason, in addition to domestic variables, exogenous assumptions about dynamics in European GDP will be included, the transmission of which to domestic GDP is also modelled using time-varying parameters. The inclusion of foreign variables and the explicit modelling of a small open economy are the main contribution to the existing literature on the forecasts of GDP using TVP models.

Six models with time-varying parameters (one regression and five VAR models) are recursively estimated and forecasted in this paper. All of the models are also estimated with fixed parameters using the least squares method (OLS), while a naive forecast is evaluated additionally. A four-quarters-ahead forecast for domestic GDP is produced for each iteration and each model. Several standard indicators of the mean forecast error are calculated for all models. The Diebold-Mariano statistical test (Diebold and Mariano, 1995) is additionally carried out to compare the forecast performances of the models with time-varying parameters with the performances of the fixed parameter models.

The results indicate that the mean forecast error for most of the models with time-varying parameters is smaller than for the equally specified fixed parameter models, and for the naive model. It is worth stressing that the use of the TVP model improves forecasts by 1–15% in relation to the simple naive model, which means that the marginal contribution of the use of such models is very modest. Despite this, such results are not surprising if we take into consideration the results obtained in the mentioned papers for other countries in which improvement of GDP forecasting is within the same range. The results of this paper together with the results from other similar papers indicate that GDP is a variable which is very difficult to forecast by atheoretical econometric models for horizons exceeding one quarter, even in the case of sophisticated models that take structural changes into account. However, by comparing the fixed parameter model and the model with time-varying parameters, an important result is obtained. On the sample used, the forecast error for almost all models with time-varying parameters is smaller than those obtained by using the equivalent fixed parameter models, which confirms the advantage of using the model with time-varying parameters for the purpose of forecasting GDP during times of structural changes. In addition, according to the results obtained, by averaging the forecasts for all models used, the mean forecast error is lowered by several additional percentage points.

The paper is organised as follows: The second chapter explains the modelling of structural changes with reference to the existing literature, and the reasons for using such models in the forecasting of economic variables are explained. This is followed by a detailed description of the models used in this analysis and the methods of their evaluation. The fifth chapter presents the results and discussions about them, followed by the conclusion.

2 Modelling structural changes and forecasting

2.1 Modelling structural change

Over the past twenty years or so, a large number of papers have focused on the econometric modelling of structural changes similar to those mentioned in the Introduction.

Goldfeld and Qandt (1973) is the first paper on the modelling of changes or breaks in parameters, in which the so-called Markov regime switching models were developed, i.e. models with sudden changes in regression parameters. In such models, different values of parameters are identified with regard to the different regimes of the described process. Hamilton (1989) made the next step in the development of Markov regime switching models by adding dynamic or autoregressive elements to such specifications. The paper marked the beginning of a more intensified application of this type of models (Kim and Nelson, 1999).

Except in frequentist econometrics, models with time-varying parameters were also developed in the domain of Bayesian econometrics. The first papers in this field include Carter and Kohn (1994) and Kim and Nelson (1998). However, their broader application began only following the paper by Cogley and Sargent (2002), in which the Time-Varying Parameter Bayesian Vector Autoregressive model (TVP-BVAR) was developed. Unlike most of the earlier papers in which sudden changes in parameters were modelled, the mentioned VAR model enables the modelling of smooth continuous changes in parameters. An additional advantage of such models is their flexible structure in which the process according to which every single parameter changes is independent of the processes of all other parameters. In that work, a monetary TVP-BVAR is estimated, in conjunction with a structural analysis, however the same models are also subsequently used for the forecasting of macroeconomic variables. For example, Canova (2002) explores the predictive ability of a large number of models with fixed and time-varying parameters, and concludes that the TVP-BVAR models improve forecasts of inflation substantially, and that such models are very good at identifying the changes in the direction of inflation.

2.2 Forecasting economic activity

Following the onset of the economic crisis in 2008, economists are often criticised about standard econometric and macroeconomic models being incapable of foreseeing such strong recessions. For this reason precisely, in recent years many papers have been published in which the predictive performances of alternative methods for the forecasting of GDP and other indicators of economic activity are examined. Models with time-varying parameters, primarily TVP-BVAR models, also play an important role. D'Agostino et al. (2011) forecast unemployment (as a measure of economic activity), inflation and interest rates for the US, using TVP-BVAR, and conclude that the mentioned model, on average, gives much better forecasts than any other estimated fixed parameter models, but they also state that the forecasts of the unemployment rate are much worse than the forecasts of the other two variables. Mumtaz et al. (2012) explore forecast performances of the same set of variables as in the previously mentioned paper for Great Britain, but instead of unemployment, they use GDP as a more adequate measure of economic activity. In addition to the standard TVP-BVAR model, they also estimate a broad set of other Bayesian models with time-varying parameters as well as different specifications of Markov regime switching models. In that paper, a large number of models (25) were evaluated on such a large sample (1976 – 2007) that the results obtained can be considered relevant. The basic conclusion is that, in general, macroeconomic variables are better forecasted using TVP models, when compared with simple benchmark models, fixed parameter models, but also compared with the regime switching models that show the worst performances. Again, the results indicate that it is much easier to "beat" simple naive forecasts for inflation and interest rate than for GDP. Improvements of GDP forecasts using TVP models compared with the AR(1) model are around 5%, depending on the model used. Marcellino et al. (2012) carry out an analysis similar to that in Mumtaz et al. (2012) with a broader set of variables and a larger number of countries (all OECD countries). However, the paper exclusively evaluates the forecasts for univariate models with time-varying parameters, and conclusions vary depending on the variables and countries that are analyzed. The results indicate that all variables that describe economic activity, including GDP, show somewhat poorer performances, so that mean forecast errors for the TVP models are sometimes even bigger than those for the simple fixed parameter autoregressive models.

In general, the results obtained in the mentioned papers vary, but it is clear that GDP forecasts using the TVP model are only marginally better than the forecasts of the simplest naive forecasts. The possible improvement of a forecast or the lowering of the mean error when using models with time-varying parameters in relation to the simple naive models ranges from around 5% to a maximum of 15%, while forecast errors in using the fixed parameters VAR models or regime switching models are even bigger than those obtained by the naive models. All of the aforementioned leads to the conclusion that GDP is a variable that is difficult to forecast, even for short-term horizons, so that the facts presented above should be borne in mind when interpreting the results of this paper.

If the mentioned sophisticated models cannot improve forecasts substantially, it is reasonable to raise the question of which methodology would be adequate for forecasting GDP. One of the possible responses is forecasting using a combination of different models and expert judgements. However, Croatia's GDP forecasts from this analysis are not comparable with the professional forecasts of different institutions, such as the Croatian National Bank, the Institute of Economics, the International Monetary Fund or the European Commission, because the time horizon, the frequency, the recurrence of forecasts and the availability of data in real time are not comparable. However, it can be assumed that such professional forecasts are better than the purely model-based forecasts because they can use a broader information set, and, in addition to atheoretical model-based forecasts, different structural models as well as expert judgements are taken into account. Precisely for this reason, the models tested here can serve exclusively as one of the inputs in a broader GDP forecasting process. It is also worth mentioning that in this paper the examples of only a few models with time-varying parameters are presented, while in practice the set of models and variables can vary and be complemented at every projection iteration. Apart from the mentioned models for the so-called short-term forecast of economic activity, models for early estimates of current quarter GDP, or nowcasting models, also play an important role both in the literature and in application. By using such models, forecast errors are significantly smaller than for naive models, or any other econometric model (Reichlin et al., 2006, Marcellino and Schumacher, 2007, and, for Croatia, Kunovac and Špalat, 2014). The results on the state of the economy in the current quarter obtained by such nowcasting models usually serve as the starting point for forecasting economic activity for the future period using atheoretical or more complex structural models. When using the kind of models evaluated in this paper in practical applications one can also use GDP nowcasts as the input for the first quarter of the forecasting horizon. However, this paper does not aim to explain the forecasting process in practice, so that available monthly indicators in the first quarter of the forecasting horizon will be ignored in all used models.

3 Forecasting models

In this chapter, first the models with time-varying parameters used in this paper are described in detail, as well as the manner of estimation of these models. After that a brief description of benchmark models is given.

3.1 Time-varying parameter vector autoregressive models

The time-varying parameter vector autoregressive (VAR) model that will be used for forecasting Croatian GDP is similar to the models in Cogley and Sargent (2002) and Mumtaz et al. (2012). This paper will estimate five TVP-BVAR models with a different set of variables. The simplest model includes only GDP and loans as endogenous variables, while no exogenous variables are included in the model. The second model, in addition to the two mentioned variables, includes the European Union's GDP as an exogenous variable. The three additional TVP-BVAR models consist of three endogenous and one exogenous variable.[2]

The European Union's GDP is the exogenous variable for all three mentioned models, while endogenous variables for one of the models include GDP, loans and the Zagreb Stock Exchange Index (CROBEX), while

2 In this paper, a VAR with a maximum of four variables is used because the used sample is too short for any larger number of variables. For the same reason, one lag is used for all VAR models.

for the second one they are GDP, loans and credit default swap (CDS) for the Croatian sovereign bond, and for the third model they are GDP, loans and the interest rate on short-term loans. The variables: GDP, loans, CROBEX and EU GDP are included in the model in the annual rates of change, while CDSs are included in levels. All models include loans because the pre-crisis economic growth was, to a large extent, based on the increase in credit activity, while time-varying parameters enable the modelling of a gradual weakening of the importance of credit activity for the growth of GDP after 2008. The Zagreb Stock Exchange Index is included in one of the models to take into consideration the correlation between the domestic financial market and real economic activity. It is interesting to model the mentioned correlation using time-varying parameters because it differs greatly in turbulent and calmer times (Kunovac 2011). In addition, several papers, such as Fischer and Merton (1984), Guo (2002) or Barro (1990) propose a series of theoretical and empirical reasons because of which the price of shares should contribute to forecasting real economic activity. It can be assumed that the price of borrowing will also have an impact on GDP, and in addition to domestic factors, foreign factors also affect the borrowing cost. For this reason, CDS is used in one of the models.

For all of the five models, TVP-BVAR is estimated in its reduced form:

$$Y_t = c_t + \mathbf{B}_t Y_{t-1} + D_t x_t + u_t \tag{1}$$

where Y_t is a vector of endogenous variables, Y_{t-1} is a vector of endogenous variables with a lag 1 x_t is the exogenous variable, c_t, $\mathbf{B}_t$ and D_t are parameter matrices (vectors), while u_t is a vector of the residuals. Vectors Y_t, c_t and D_t are dimensions $N\times 1$, where N is the number of endogenous variables (equations), and $\mathbf{B}_t$ is the matrix of dimension $N\times N$. The difference between equation 1 and a standard fixed parameters VAR consists exclusively of a time index t being ascribed the parameters. If it is assumed that the equation (1) is the measurement/observation equation of the model written in the state-space representation, parameters c_t, $\mathbf{B}_t$ and D_t will be unobserved variables, and u_t the measurement error. For easier writing, all elements from matrices c_t, $\mathbf{B}_t$ and D_t will be combined in a vector β_t, dimension $(N\times(N+2))\times 1$. Unobserved variables or time-varying parameters written in this form will be modelled by the state equation or the first-order autoregressive process:

$$\beta_t = \mathbf{F}\beta_{t-1} + v_t \tag{2}$$

As common in the existing literature[3], it will be assumed that parameters follow a random walk process, or that matrix $\mathbf{F}$ is an identity matrix. Residuals u_t and v_t have the common features of non autocorrelation, with expectation zero and homoscedastic or constant variances

$$E(u_t) = 0, \operatorname{cov}(u_t, u_s) = 0, \text{ for } t \neq s \tag{3}$$

$$var(u_t) = \mathbf{R} = \begin{pmatrix} R_{1,1} & 0 & \cdots & 0 \\ 0 & R_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & R_{N,N} \end{pmatrix} \tag{4}$$

$$E(v_t) = 0, \operatorname{cov}(v_t, v_s) = 0, \text{ for } t \neq s \tag{5}$$

$$var(v_t) = \mathbf{R} = \begin{pmatrix} Q_{1,1} & 0 & \cdots & 0 \\ 0 & Q_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & Q_{(N\times(N+2)),(N\times(N+2))} \end{pmatrix} \tag{6}$$

$$\operatorname{cov}(u_t, v_t) = 0 \tag{7}$$

3 See, for example, Cogley and Sargent (2002) or Mumtaz et al. (2012).

In this case, $(T\times N\times(N+2))$ time-varying parameters and the additional N diagonal elements of matrix $\mathbf{R}$ and $(N\times(N+2)$ of the diagonal elements of matrix $\mathbf{Q}$ (fixed parameters) will be estimated. It is worth stressing that all parameters of the TVP-BVAR models, as well as TVP regressions, will be estimated via Bayesian simulation. For this purpose, the Carter & Kohn algorithm is used, as described in more detail in Appendix 1.

3.2 Regression with time-varying parameters

Regression with time-varying parameters is the simplest TVP model used for forecasting GDP in this paper. In this regression, the annual Croatian GDP growth rate (Y_t^{GDP}) is regressed on the annual EU GDP growth rate (Y_t^{GDPEU}), the annual Croatian GDP growth rate from the previous quarter (Y_{t-1}^{GDP}) and a constant (c_t).

$$Y_t^{GDP} = c_t + \beta_t^{lag} Y_{t-1}^{GDP} + \beta_t^{EU} Y_t^{GDPEU} + u_t \tag{8}$$

The above written equation is a measurement/observation equation, while parameters, as in the case of TVP-BVAR, will be unobserved variables, and u_t the measurement error. The state equation for parameters β_t is also described by the random walk process ($\mathbf{F} = \mathbf{I}$), without a constant, or by equation (2).

For the state (2) and measurement (8) equation, the residuals (u_t, v_t) are mutually uncorrelated, non autocorrelated and have expectation zero and constant variances R and $\mathbf{Q}$. Consequently, in the model described above, except for time-varying parameters, or unobserved variables, it is also necessary to estimate three fixed parameters from the matrix $\mathbf{Q}$ and the parameter R.

3.3 Bayesian parameter estimation

All parameters from the described TVP models will be estimated with the use of Bayesian methods, which means that it is assumed that each parameter is a random variable, unlike the frequentist approach, in which it is assumed that only the parameter estimate, and not the parameter itself, is a random variable.[4] In Bayesian econometrics, posterior distributions will be parameter estimators obtained as a combination of the likelihood function and the prior distribution. Usually models are used in which posterior distributions are of the same type as prior distributions. Such distributions are referred to as conjugated. However, for the state space model class, as well as for many other models, the posterior distribution is not necessarily of the same type as the prior distribution. In addition, in such models, posterior distributions are often difficult to derive analytically, so that they are known only up to the normalising constant. However, it is possible to approximate the posterior distribution for such cases using the Monte Carlo Markov Chains (MCMC) simulation methods. MCMC methods enable the simulation of a Markov Chain, whose limiting distribution is equal to the distribution of interest. After a sufficiently large number of simulations, certain conditions being satisfied[5], the numerical simulation can be considered as a sample from the posterior distribution of interest. The Metropolis-Hastings algorithm is the most frequently used MCMC method in Bayesian econometrics. If it is possible to derive conditional distributions analytically for each parameter (conditionally on numerical values for all other parameters) for a model, the Gibbs sampler will be used, which is a special case of the Metropolis-Hastings algorithm. Briefly, the Gibbs sampler is an MCMC algorithm which enables the approximation of marginal or joint posterior distributions of interest, by numerical simulation from conditional distributions.

Carter and Kohn (1994) derive conditional posterior distributions analytically for unobserved variables in state space models and develop an algorithm which enables the inclusion of the Gibbs sampler in such models.

4 The following section only briefly explains the basic idea of the Bayesian estimation and the Gibbs sampler, while a more detailed explanation is given in Kunovac (2012).

5 See Casella and George (1992) and Robert and Casella (2004) for the necessary conditions for the convergence of the Gibbs sampler.

Below, the basic idea of the mentioned algorithm is briefly explained, and a detailed description is given in the Appendix to the paper.

To begin the simulation process using the algorithm of Carter and Kohn, initial numerical values and prior distributions for all unknown parameters and unobserved variables should be determined in Step 0. The elements of the variance-covariance matrix of the residual of the measurement equation, **R**, are distributed *a priori* according to the inverse Wishart distribution, $p(\mathrm{R}) \sim IW(\underline{\mathrm{R}}, \underline{\mathrm{s}})$, where $\underline{\mathrm{R}}$ and $\underline{\mathrm{s}}$ denote the scale parameter and the number of the degrees of freedom of the prior distribution. For the elements of the variance-covariance matrix of the residual of the state equation, **Q**, a prior inverse Wishart distribution is also assumed, $p(\mathrm{Q}) \sim IW(\underline{\mathrm{Q}}, \underline{\mathrm{T}})$, while for the unobserved variables a normal prior, $p(\beta) \sim N(\underline{\mathrm{b}}, \underline{\mathrm{E}})$ is assumed. After priors are determined and arbitrary initial values selected, the following two steps of simulation from conditional posterior distributions will be iterated, using the Gibbs sampler for M iterations:

1 Conditional on observed data and numerical values of the hyperparameters[6] in $\mathbf{R}^{(m-1)}$ and $\mathbf{Q}^{(m-1)}$ the values for unobserved variables $\tilde{\beta}_T^{(m)}$ are generated.[7]

2 Conditional on observed data and numerical values for $\tilde{\beta}_T^{(m)}$ the values for hyperparameters are generated.

For the first step in the first iteration the algorithm begins with the simulation for the series of unobserved variables, $\tilde{\beta}_T^{(1)}$, conditional on initial hyperparameter values from the matrices $\mathbf{R}^{(0)}$ and $\mathbf{Q}^{(0)}$, while for all other iterations $m=2,\ldots M$ time-varying parameters $\tilde{\beta}_T^{(m)}$ are simulated using the information on the hyperparameters from the previous iteration. The process of simulation from the first and the second step will continue iteratively until the convergence of distributions to the final stationary posterior distributions, and the statistical analysis will be conducted on the latest $M-K$ iterations, where the first K iterations are the so-called "burn-in". For all Bayesian models in this paper the values $M=30000$ and $K=15000$ are selected.

3.3.1 Specification of prior distributions and initial values

The selection of the parameters of prior distributions will be unbiased enough to take into account an equal information set in the estimation and forecasting of each model. For this reason, the parameters of prior distributions are simply the OLS estimates of the parameters of the VAR or regression model that are obtained on a training sample.[8] The values from the variance-covariance matrix of the residuals of the measurement equation, estimated on a training sample, will be used as a prior for $\underline{\mathrm{R}}$, and will also be used to compute the prior for the variance-covariance matrix of the measurement equation, $\underline{\mathbf{Q}}$. According to Cogley and Sargent (2002), $\underline{\mathbf{Q}}$ will be defined as $\underline{\mathrm{Q}} = \mathbf{Q}^{T_0} \times T_0 \times \omega$, where $\mathbf{Q}^{T_0}$ is the OLS estimate for Q, T_0 is the number of the quarters included in the training sample, and ω is a scalar whose numerical value can be set arbitrarily. It is worth stressing that for the final result in the form of *a posteriori* time-varying parameters the value of *a priori* parameters in the matrix $\underline{\mathrm{Q}}$ is very important because they will be used for controlling the degree of variation of the time-varying parameters. Consequently, the selection of values for the scalar ω will play a very important role. Although in Cogley and Sargent (2002) the value of 0.00035 is used, in this paper it will be considerably higher because for a relatively short sample, or a small T_0, a sufficiently large variation for $\bar{\beta}_t$ cannot be achieved. For this reason, for each of the models used, a sufficiently large value ω was selected, for which there is an observable variation for $\bar{\beta}_t$ that will not lead to numerical problems in simulation and will converge the simulated values to the final stationary posterior distributions within the first K iterations. The values for ω, depending on the model, range from 0.05 to 0.15.[9] The number of the degrees of freedom of the prior distributions for the hyperparameters, $\underline{\mathrm{s}}$ and $\underline{\mathrm{T}}$, equals T_0. The mean of the prior distribution of unobserved variables, $\underline{\mathrm{b}}$, is equal to the corresponding OLS estimate obtained on the training sample. All initial values are equal to the priors described above.

6 In this case, as in D'Agostino et al. (2010), the common term of "hyperparameters" is used for **R** and **Q** to distinguish them from the parameters of the VAR model $\tilde{\beta}_T^{(m)}$, or time-varying parameters.

7 For simplicity, the time series of the unobserved variables $\{\beta_t\}_{t_0}^{T}$ will be denoted as $\tilde{\beta}_T$. In the same way, the series of endogenous and exogenous observed variables will be denoted as $\tilde{\mathbf{Y}}_T$ or $\tilde{x}_T$.

8 The test sample covers the first 20 quarters of the entire sample.

9 Models with more variations, or higher ω, have shown much smaller forecast errors than the models in which ω is relatively lower.

3.4 Benchmark forecasts

This chapter describes several fixed parameter models that are estimated and forecasted on the same moving sample to compare the mean forecast errors of such models with the TVP models. For this purpose, two groups of models are used: naive models, as they are called, as well as regressions and VAR models with fixed parameters which are identically specified as their TVP counterparts.

Naive forecasts are usually used for simple forecasting of a given time series, without any additional information and parameter estimation. The random walk process (RW) and the unconditional mean forecast (UCM) are the models commonly referred to in the literature. In addition to the two mentioned naive models, the predictive abilities of other standard benchmark models were tested, but the UCM forecast, using growth rates for the last four quarters, produced the smallest forecast errors, and for this reason it was used as the adequate benchmark forecast.

The GDP forecast using the UCM for each horizon h is defined as the average of the last four observed GDP growth rates:[10]

$$\Delta\hat{Y}_{T+h} = \frac{1}{4}\Delta Y_T + \frac{1}{4}\Delta Y_{T-1} + \frac{1}{4}\Delta Y_{T-2} + \frac{1}{4}\Delta Y_{T-3} \tag{9}$$

Fixed parameter VAR models and regression that serve as an additional benchmark for the evaluation of the TVP model are estimated with the OLS method using the same number of lags and equal endogenous and exogenous variables as the respective TVP model. The aim of including these models in the analysis is to obtain a direct comparison of forecast performances of Bayesian TVP models and standard classical fixed parameter models with identical specifications, which means that the difference in forecasts results exclusively from the difference in the parameter estimation method.

4 Evaluation of forecasts

All of the described models are estimated and forecasted recursively on a moving sample of 20 quarters. Hence, all forecasted GDP growth rates for all models and all 20 iterations for the forecasting horizon of four quarters will be used for the purpose of forecast evaluation and model comparison. For this purpose, a dataset for the period from the first quarter of 2000 to the third quarter of 2013 was used. For the first iteration, the models were estimated on a sample up to 2007Q4, and the annual GDP growth rates for four quarters, i.e. for 2008Q1 – 2008Q4 were forecasted. The obtained forecasts were compared with the realized values for the mentioned period and forecast errors for each of the four forecasted quarters were calculated. Subsequently, the sample for model estimation was expanded by one quarter, or until 2008Q1 and GDP growth rates for all of the models were forecasted for the period of 2008Q2 – 2009Q1, while forecast errors were calculated in the same manner as for the previous iteration. In this way, the process of successive expansion of the sample continued until the last quarter, i.e. the third quarter of 2012, so that the last forecast refers to the period of 2012Q4 – 2013Q3.[11]

The forecast error was calculated from all obtained forecasts and for each model used, while four series of 20 errors for each model were used for the calculation of a number of indicators. First, two common

10 The unconditional mean forecast refers to the quarterly rates of seasonally adjusted GDP because such forecasts have shown much smaller mean forecast errors than those forecasted using annual growth rates. If the above benchmark forecasts were calculated using annual growth rates, the obtained mean squared forecast errors would be by around 30% higher than on those calculated using quarterly growth rates. For this reason, guided by the criterion of using the best benchmark forecast, the unconditional mean forecasts were used, obtained from the quarterly growth rates of seasonally adjusted GDP. To compare naive forecasts with the forecasts of other models (estimated at the annual growth rates) forecasted levels of seasonally unadjusted GDP were created from the quarterly rates mentioned. Finally, this unadjusted forecasted series of GDP is used to produce the necessary annual growth rates.

11 2013Q3 refers to the last quarter in which GDP data were available at the time when the paper was drafted.

measures were estimated: the root mean squared forecast error (RMSFE) and the mean absolute forecast error (MAFE).

If we assume that $\{\hat{Y}_{t+h}\}_1^T$ is a series of GDP forecasts for the h^{th} period ahead, and $\{Y_{t+h}\}_1^T$ is the respective realizations, the RMSFE for the h horizon can be written, as follows:

$$RMSFE_h = \sqrt{\frac{\sum_{t=1}^{T}(\hat{Y}_{t+h} - Y_{t+h})^2}{T}} \tag{10}$$

while the MAFE for h horizon is calculated, as follows:

$$MAFE_h = \frac{\sum_{t=1}^{T}|\hat{Y}_{t+h} - Y_{t+h}|}{T} \tag{11}$$

In this specific case $T = 20$, and h = {1,2,3,4}, which means that four RMSFEs and MAFEs will be calculated for each model obtained from a series of 20 forecast errors.

In addition to the above simple measures of mean forecast errors, a statistical test for the comparison of forecasts of two different models should also be carried out. For this purpose, the Diebold-Mariano test (Diebold and Mariano, 1995) was used. This test has become a common comparison measure in the existing literature on economic forecasts.[12] In this paper, the mentioned test will be used for the comparison of forecasts for each model with time-varying parameters with forecasts of the respective fixed parameter model, and it will be determined whether, statistically, the TVP model differs from the identically specified fixed parameter model.

4.1 Data

The data used to estimate the models in this paper cover the period from the first quarter of 2000 to the third quarter of 2013. The measure of domestic economic activity consists of real GDP[13], or annual rates as its stationary transformation. The credit series used is the stock of total loans at the end of the observed quarter, and the mentioned series is also transformed into the annual rates of change. The average quarterly value of the Zagreb Stock Exchange Index is also included in the models in the form of annual rates of change. The two remaining endogenous variables in this paper include the credit default swaps (CDSs) for the five-year sovereign bond of the Republic of Croatia and the average interest rate on loans up to one year. Both series are stationary and are therefore included in the models in levels. The CDS and CROBEX time-series are taken from the Bloomberg terminal, and the interest rate and loans are taken from the database of the Croatian National Bank.

The annual EU real GDP growth rate was used as a comparable measure of foreign economic activity.[14] The assumptions about the forecasts of the annual European GDP growth rate are taken from the publication *Consensus Economic Forecast*. To approximate the information set available in real time in each iteration of recursive forecasting, the forecasts from this publication were taken for each first month in the second quarter of the forecasting horizon. For this reason, for the forecasting of Croatian GDP, starting from the first, second, third and fourth quarter of each year, the forecasts of European GDP available in the issues of *Consensus Economic Forecast* for April, July, October and January respectively were used. The basic purpose of such an approach to using external assumptions is that domestic GDP in each iteration is forecasted with the most recent available forecast of foreign GDP. For instance, for the latest forecast iteration, the parameters of the model are estimated on a sample until 2012Q3, while domestic GDP is forecasted for the period of 2012Q4 – 2013Q3. In this case, the forecast of European GDP that is used refers to the same period, i.e. 2012Q4 – 2013Q3,

12 The Diebold-Mariano test is briefly described in Appendix II.

13 Gross domestic product, fixed prices, in the prices of the previous year, real growth rates according to the same quarter of the year earlier, are taken over from the First Release number 12.1.1/3 of the Croatian Bureau of Statistics.

14 The data for actual EU GDP are comparable with those for Croatian GDP, and have been taken over from the Eurostat database.

published in the *Consensus Economic Forecast* in January 2013. It was precisely in January 2013 that domestic GDP was available in real time until 2012Q3, or GDP up to the last quarter included in the sample for the parameter estimation.

5 Results

Two basic indicators of mean forecast errors for all models used for the forecasting horizon from the first to the fourth quarter are shown in the tables below. The first table shows the results for the first measure, or RMSFE, while the second table shows the results for MAFE. All results in tables 1 and 2 are shown in relation to the naive benchmark forecast in such a way that numbers less than one denote mean errors of the model forecasts which are smaller than the corresponding mean errors of the naive forecast.

The results in the first table clearly indicate that for the forecasts of the TVP model, of 6×4 relative RMSFEs, as many as 22 are less than one, while only two numbers are equal to or higher than one, which means that, by using a squared loss function, the forecast is more precise than the unconditional mean forecast for all forecasting horizons and for almost all of the TVP models. On the other hand, for the fixed parameter models, only four models had the RMSFE less than one. By comparing the forecasts of TVP models with the forecasts of the fixed parameter models, it is possible to conclude that for all models the forecasts for the TVP models are more accurate. The results in Table 2 additionally confirm the presented conclusions. According to this table, in 23 cases the mean absolute forecast error of the TVP models is smaller than the error of the fixed parameter models. The results obtained are similar to those for the US in D'Agostino et al. (2010), where the forecast error of economic activity obtained with the TVP-BVAR model for the first quarter is similar to the mean error of the naive forecast, while the error for the fourth quarter is almost 20% smaller than that of the

Table 1 Root mean square forecast error (RMSFE) in relation to benchmark model

Horizon	1	2	3	4
Models:				
TVP-reg	1.22	0.98	0.92	0.85
TVP-BVAR2	0.99	1.00	0.95	0.89
TVP-BVAR3	0.92	0.95	0.94	0.87
TVP-BVAR4cds	0.99	0.99	0.97	0.89
TVP-BVAR4cbex	0.94	0.95	0.91	0.85
TVP-BVAR4kam	0.93	0.96	0.94	0.87
Fix-reg	1.25	1.27	1.20	1.06
Fix-VAR2	1.29	1.39	1.38	1.30
Fix-VAR3	1.10	1.01	1.01	1.01
Fix-VAR4cds	1.07	1.02	0.97	0.90
Fix-VAR4cbex	1.01	1.03	1.00	0.93
Fix-VAR4kam	1.09	1.05	1.00	0.91
TVP	0.85	0.89	0.88	0.84
Fix	0.89	0.97	0.95	0.88
Total	0.79	0.90	0.89	0.86

Note: Prefix "TVP" denotes the models with time-varying parameters, and "Fix" denotes the fixed parameter models. "Reg" denotes regression, "VAR2" denotes the VAR with two endogenous variables, the "VAR3" model with two endogenous and one exogenous variable, while models denoted with "VAR4" represent the VAR with three endogenous and one exogenous variable. Suffixes "cds", "cbex" and "kam" are CDS, CROBEX or interest rate, respectively, or the variable which differs for the individual VAR4 model. The line under the name of the variable denotes averaged forecasts.
Source: Author's computation.

Table 2 Mean absolute forecast error (MAFE) in relation to benchmark model

Horizon	1	2	3	4
Models:				
TVP-reg	1.19	1.04	1.05	0.95
TVP-BVAR2	1.01	0.98	0.93	0.92
TVP-BVAR3	0.93	0.83	0.89	0.94
TVP-BVAR4cds	1.07	0.89	0.91	0.96
TVP-BVAR4cbex	0.95	0.83	0.91	0.94
TVP-BVAR4kam	0.97	0.83	0.89	0.95
Fix-reg	1.25	1.28	1.33	1.20
Fix-VAR2	1.19	1.36	1.52	1.47
Fix-VAR3	1.04	0.92	0.99	1.08
Fix-VAR4cds	1.10	0.93	0.96	0.95
Fix-VAR4cbex	0.97	0.95	0.98	0.99
Fix-VAR4kam	1.04	0.96	1.03	1.06
TVP	0.90	0.77	0.81	0.86
Fix	0.91	0.79	0.81	0.86
Total	0.84	0.77	0.77	0.82

Note: Prefix "TVP" denotes the models with time-varying parameters, and "Fix" denotes the fixed parameter models. "Reg" denotes regression, "VAR2" denotes the VAR with two endogenous variables, the "VAR3" model with two endogenous and one exogenous variable, while models denoted with "VAR4" represent the VAR with three endogenous and one exogenous variable. Suffixes "cds", "cbex" and "kam" are CDS, CROBEX or interest rate, respectively, or the variable which differs for the individual VAR4 model. The line under the name of the variable denotes averaged forecasts.
Source: Author's computation.

naive forecast. In the case of Croatia, tables 1 and 2 show a gradual decrease in relative forecast errors as the forecasting horizon increases. An additional similarity with the results in the mentioned paper is that models with time-varying parameters on average give better forecasts than the equally specified fixed parameter models. The paper by Mumtaz et al. (2012) also leads to the conclusion that the best forecasts of economic activity for the United Kingdom are obtained using Bayesian TVP models. It is a general conclusion that the relative forecast errors obtained in this paper do not diverge significantly from the relative forecast errors obtained by the TVP models for other countries.

The results of the individual models indicate that the smallest forecast errors are produced by the TVP-BVAR model that includes GDP, loans and foreign GDP ($TVP-BVAR3$), as well as the TVP-BVAR model that includes GDP, loans, CROBEX and foreign GDP ($TVP-BVAR4cbex$). The results justify the inclusion of the Zagreb Stock Exchange Index as a relevant high-frequency measure for forecasting real economic developments, as confirmed and explained in Barro (1990). On the other hand, the biggest forecast errors in the class of VAR models with time-varying parameters are produced by the TVP-BVAR model that includes GDP, loans, foreign GDP and CDS ($TVP-BVAR4cds$). The results related to the models with the CDS premium are not surprising because the CDS premium in a larger portion of the sample had a very small variance with only a mild correlation to GDP. The sovereign debt market and the relevance of the CDSs for the explanation of economic developments gained in importance only with the outbreak of the so-called fiscal crisis of 2011[15], while a larger portion of the sample analysed here refers to the period before 2011. However, it can be expected that in the future, as a result of the excessive growth in public debt and the related pressure on a further change in fiscal policy, the country risk premium may become more relevant in an explanation of GDP movements. In addition to the $TVP-BVAR4cds$ model, regressions ($Fix-reg$ and $TVP-reg$) also produce slightly bigger forecast errors. A similar result is obtained in D'Agostino et al. (2010), where VAR models with time-varying parameters give more precise forecasts than the simpler regressions and the ARIMA models with time-varying parameters.

15 For more detail on CDS premiums and government bond yields see Kunovac (2013).

In the introduction to the paper it is emphasised that one of the main contributions of this paper is the modelling of a small open economy using the TVP models, so that it is necessary to compare the results for $TVP-BVAR2$ and $TVP-BVAR3$ models, in particular because two models differing by only one variable, foreign GDP, are concerned. The results indicate that for all forecasting horizons the mean forecast error is smaller for the forecasts using the $TVP-BVAR3$ model, which justifies modelling the Croatian economy as a small open one for the purpose of GDP forecasting.

So far, results for the individual models have been commented on, and hereinafter we give a brief overview of the results for averaged forecasts. These results are presented in the bottom sections of both tables so that first the mean forecast errors are shown for the models with time-varying parameters, then for the fixed parameter models and, finally, for all models used in the bottom row of the table. The basic conclusion resulting from the mentioned indicators is the following: mean errors produced using the average of the forecasts of different models are considerably lower than those produced by individual models. The results also indicate that the difference between the fixed parameter models and the models with time-varying parameters is smaller if the mentioned average forecasts are taken into consideration. However, it is still true that forecasts for the models with time-varying parameters are slightly more precise. For the second and the third horizon the mean error for the TVP models is the smallest, while for the other horizons the smallest error is the error for the forecast of the average of all models, if the results for RMSFE are observed. In the case of MAFE, the results confirm the forecast by total average as the best among the forecasts used. The results that show the advantage of averaging are also confirmed by some similar surveys, such as Hendry and Clements (2001).

Table 3 Diebold-Mariano test results for the comparison of models with time-varying parameters and fixed parameter models

Horizon	1	2	3	4
Models:				
Reg	–0.19	–1.46	–2.16**	–1.89*
VAR2	–1.33	–1.91*	–1.5	–1.41
VAR3	–0.76	–0.53	–1.14	–1.51
VAR4cds	–0.75	–0.63	–0.59	–1.11
VAR4eq	–0.73	–0.88	–1.05	–1.1
VAR4kam	–1.51	–1.26	–2.37**	–2.51**
Average	–0.35	–0.72	–0.89	–1.07

Note: The asterisks ** and * denote the difference between the two models for the of 5% or 10% significance level. The prefix of the test-statistics is negative if forecast errors of the analysed model with time-varying parameters are smaller than the forecast errors of the relevant fixed parameter model.
Source: Author's computation.

In addition to the comparison of the models using RMSFE and MAFE, the Diebold-Mariano statistical test was also conducted, testing whether, statistically, the series of forecast errors of the TVP models differs significantly from the set of forecast errors of the fixed parameter models. The test was conducted for all individual models, as well as for the average of the models. The results are shown in Table 3 in the form of test statistics. The results indicate that in 5 out of the 28 tests the TVP models, with a 10% significance level, produce fewer errors than the fixed parameter models. In addition, it is obvious that all values for the statistics shown have a negative sign, so that a large number of test-statistics ranges from –1.5 to –1, which means that by using the standard 10%, a large number of test results are only marginally insignificant. For instance, by using the 20% limit, the series of forecast errors for the TVP models is statistically significantly smaller than the errors of the fixed parameter models for 11 out of the 28 conducted Diebold-Mariano tests. If the limit of one standard deviation is considered, as many as 17 tests significantly confirm relatively fewer errors of the TVP models.

Despite the fact that all of the results given clearly indicate that forecasts for the TVP models are more precise than the forecasts of the fixed parameter models, a potential limitation with regard to the length of the time series used should be emphasised. Because the sample was too small, relatively few forecast iterations (20 iterations) were carried out in this paper, and it is known that the strength of the statistical test used is weak on small samples (Harvey, 1997). However, the maximum possible number of forecast iterations is twenty at this moment, taking into consideration all the limitations described in the previous chapters of the paper.

6 Conclusion

The main goal of this paper was to examine whether short-term forecasts of Croatian GDP are more precise if structural changes are modelled explicitly using time-varying parameters. It is the models with time-varying parameters that are imposed as a logical choice for forecasting Croatian economic activity because the recent economic crisis has led to considerable changes in the country's economy. For this reason, several Bayesian models with time-varying parameters are posited in this paper, the forecasts of which are compared with those of equally specified fixed parameter models and the naive benchmark forecast. In order to take into consideration the specificities of a small open economy, in addition to domestic variables, exogenous assumptions on European GDP developments are also included in the mentioned models. The results are evaluated in the customary manner by comparing forecast errors obtained by iterative forecasting on a moving sample. The mentioned errors are obtained by the evaluation of twenty iterations on a forecasting horizon of four quarters. The forecasts are compared using simple descriptive methods, that is, the standard indicators of mean forecast errors. Additionally, the Diebold-Mariano statistical test was implemented.

Three basic conclusions can be drawn from the obtained results. First, the results indicate that the forecast errors of the models with time-varying parameters are smaller than the forecast errors of equally specified fixed parameter models. For several models, differences in forecast errors are statistically significantly confirmed by the Diebold-Mariano test. The mentioned result confirms the relative advantage of using the models with time-varying parameters for the purpose of forecasting Croatian GDP.

Second, the results obtained in this paper indicate that the forecasts obtained by the averaging of all models used are better than the forecasts of the individual models.

Third, GDP is a variable that is generally difficult to forecast within a short period, even if advanced econometric models are used. Forecast errors obtained by the models with time-varying parameters are only smaller than the errors of the naive models by around 10%, while on the other hand, forecast errors for most of the fixed parameter models are even larger than those of the naive forecasts. Such results, together with the results of some earlier analyses for the US, the United Kingdom and the EU, show that by using the sophisticated atheoretical econometric models it is very difficult to "beat" the simplest naive forecasts, and that the differences in the forecasts of such models from those of naive forecasts are very small. Precisely for this reason, economists face the challenging task of developing different, particularly structural economic, models, which should enable a considerable improvement in GDP forecasts.

7 References

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8 Appendix 1 Carter and Kohn algorithm

Below is a description of the Carter and Kohn algorithm for the VAR model with time-varying parameters, while it is equivalently applicable to a simpler case of the time-varying parameter regression.[16]

The first step for each iteration includes all the basic steps of the Kalman filter necessary for the estimation and smoothing of unobservable variables in the state space model, while the second step covers exclusively the numerical simulation from the conditional distributions for **R** and **Q**. Taking into account the assumption about the independence of u_t and v_t, the measurement and state equations clearly indicate that, depending on the values for $\tilde{\beta}_T^{(m)}$, the system is transformed into a set of two types of regression equations: VAR equations for Y_t and the random walk equation for β_t. In such a case, the conditional posterior distribution for hyperparametres in matrix **R** is the inverse Wishart distribution, $p(\mathbf{R} \,|\, \tilde{\beta}_T, \tilde{\mathbf{Y}}_T) \sim IW(\bar{\mathrm{R}}, \bar{\mathrm{s}})$, with $T + \underline{\mathrm{s}}$ degrees of freedom and the following scale parameter[17]:

$$\bar{R} = (Y_t - (c_t^{(m)} + \mathbf{B}_t^{(m)} Y_{t-1} + D_t^{(m)} x_t))'(Y_t - (c_t^{(m)} + \mathbf{B}_t^{(m)} Y_{t-1} + D_t^{(m)} x_t)) + \underline{\mathrm{R}} \tag{12}$$

The conditional posterior distribution for **Q** is the inverse Wishart distribution, $p(\mathbf{Q} \,|\, \tilde{\beta}_T, \tilde{\mathbf{Y}}_T) \sim IW(\bar{\mathrm{Q}}, \bar{\mathrm{T}})$ with the scale parameter

$$\bar{Q} = (\tilde{\beta}_T^m - \tilde{\beta}_{T-1}^m)'(\tilde{\beta}_T^m - \tilde{\beta}_{T-1}^m) + \underline{\mathrm{Q}}\,. \tag{13}$$

As stated above, the Kalman filter will be used for the first step of the Gibbs sampler, i.e. for the simulation from the joint conditional posterior distribution. The conditional posterior distribution for the mentioned unobserved variables can be written in the following manner:

$$p(\tilde{\beta}_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \tilde{x}_T) = p(\beta_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \tilde{x}_T) \times p(\tilde{\beta}_{T-1} \,|\, \beta_T, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, x_t) \tag{14}$$

or by a recursive rearrangement, in the following way:

$$p(\tilde{\beta}_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \tilde{x}_T) = p(\beta_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \tilde{x}_T) \prod_{t=1}^{T-1} p(\beta_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \beta_{t+1}, x_t) \tag{15}$$

Both components of the above joint conditional posterior distribution are normally distributed:

$$p(\beta_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \tilde{x}_T) \sim N(\bar{\beta}_{T|T}, \bar{\mathbf{P}}_{T|T})\,, \tag{16}$$

$$p(\beta_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \beta_{t+1}, \tilde{x}_T) \sim N(\bar{\beta}_{t|t,\bar{\beta}_{t+1}}, \bar{\mathbf{P}}_{t|t,\bar{\beta}_{t+1}})\,, \tag{17}$$

for this reason, mean and variance parameters of the mentioned conditional distribution, $\bar{\beta}_{T|T}$, $\bar{\beta}_{t|t,\bar{\beta}_{t+1}}$, $\bar{\mathbf{P}}_{T|T}$ and $\bar{\mathbf{P}}_{t|t,\bar{\beta}_{t+1}}$ are the parameters that should be estimated using the Kalman filter. For the estimation of the mentioned parameters, numerical values for **F**, **R**, **Q** are required, as well as the observed data. For this purpose, within the Gibbs sampler for every m^{th} iteration, numerical values for hyperparameters from the iteration $m-1$ will be used, as well as the observed data $\tilde{\mathbf{Y}}_T$ and $\tilde{x}_T$. For the density function of the first distribution, or for $p(\beta_T \,|\, \mathbf{Q}, \mathbf{R}, \mathbf{F}, \tilde{\mathbf{Y}}_T, \tilde{x}_T)$ to estimate mean and variance parameters, $\bar{\beta}_{T|T}$ and $\bar{\mathbf{P}}_{T|T}$, the following group of Kalman equations will be evaluated recursively, starting from $\beta_{0|0}$ and $\mathbf{P}_{0|0}$ to the final $\beta_{T|T}$ and $\mathbf{P}_{T|T}$: [18]

16 The following explanation is accompanied by the descriptions of the Carter and Kohn algorithm from Kim and Nelson (1999, pp. 189 – 208), Karlson (2012, pp. 51 – 55) and Blake and Mumtaz (2012, pp. 77 – 89).

17 In this paper, the parameters of all posterior distributions will be marked with a line above the letter.

18 Hereinafter, X_t will denote the 1×5 matrix, in which the elements of the vectors of endogenous variables Y_t are in the first three places, the fourth is the exogenous variable, while the fifth element is equal to one. Matrix $\mathbf{X}_t$ represents the matrix of dimensions $N \times (N \times (N \times 2))$ obtained as the Kronecker product from $\mathbf{I}$ and X_t, $\mathbf{X}_t = \mathbf{I} \otimes X_t$, where $\mathbf{I}$ represents the unit matrix of dimension $N \times N$.

$$\beta_{t|t-1} = \mathbf{F}\beta_{t-1|t-1} \tag{18}$$

$$\mathbf{P}_{t|t-1} = \mathbf{F}\mathbf{P}_{t-1|t-1}\mathbf{F}' + \mathbf{Q} \tag{19}$$

$$\eta_{t|t-1} = Y_t - \mathbf{X}_t\beta_{t|t-1} \tag{20}$$

$$\mathbf{f}_{t|t-1} = \mathbf{X}_t\mathbf{P}_{t|t-1}\mathbf{X}'_t + \mathbf{R} \tag{21}$$

$$\mathbf{K} = \mathbf{P}_{t|t-1}\mathbf{X}'_t\mathbf{f}_{t|t-1}^{-1} \tag{22}$$

$$\beta_{t|t} = \beta_{t|t-1} + \mathbf{K}\eta_{t|t-1} \tag{23}$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{K}\mathbf{X}_t\mathbf{P}_{t|t-1} \tag{24}$$

The first of the mentioned equations represents the linear projection of the vector of unobserved variables β for the time t by applying the information about the vector β for $t-1$, with the known values for $\mathbf{F}$. The variances of the unobserved variables, $\mathbf{P}_t$, will be projected in the same way, using the known values of the parameters $\mathbf{F}$ and $\mathbf{Q}$. The two equations serve for a simple projection of unobserved variables and the respective variances by applying the information contained in the hyperparameters $\mathbf{F}$ and $\mathbf{Q}$, known from the previous step of the Gibbs sampler. The observed data are added in the Kalman recursion only in the third step, in which projection errors are estimated for the time t, $\eta_{t|t-1}$. The mentioned errors are obtained as the difference of the observed endogenous variables, Y_t and *fit* of the measurement equation, by applying the unobserved variables, $\beta_{t|t-1}$, projected in the first step of the Kalman recursion. In the fourth step, the variance of the mentioned error, $\mathbf{f}_{t|t-1}$, is calculated using the known numerical values for the matrix of hyperparameters $\mathbf{R}$. After that, the Kalman gain, $\mathbf{K}$, used in the last two equations, is calculated. The two equations are called updating equations, because β_t and $\mathbf{P}_t$ are updated on the basis of new information available at the time t. The new information, in this case, is contained in the errors calculated earlier, $\eta_{t|t-1}$, or in the variance, $\mathbf{P}_{t|t-1}$, and are multiplied by the matrix $\mathbf{K}$.[19]

The above described Kalman recursion gives us estimates of the values for the parameters $\bar{\beta}_{T|T}$ and $\bar{\mathbf{P}}_{T|T}$ and numerical simulation will be done for $\tilde{\beta}_T$ from the conditional distribution $p(\beta_T|\mathbf{Q},\mathbf{R},\mathbf{F},\tilde{\mathbf{Y}}_T,\tilde{X}_T) \sim N(\bar{\beta}_{T|T},\bar{\mathbf{P}}_{T|T})$. The values will be used in the process described below, which is necessary for the estimation of mean and variance parameters for $p(\beta_T|\mathbf{Q},\mathbf{R},\mathbf{F},\tilde{\mathbf{Y}}_t,\beta_{t+1},X_t) \sim N(\bar{\beta}_{t|t,\tilde{\beta}_{t+1}},\bar{\mathbf{P}}_{t|t,\tilde{\beta}_{t+1}})$. For this purpose, the following group of equations will be evaluated recursively:

$$\eta^*_{t+1|t} = \beta_{t+1} - \mathbf{F}\beta_{t|t} \tag{25}$$

$$\mathbf{f}^*_{t+1|t} = \mathbf{F}\mathbf{P}_{t|t}\mathbf{F}' + \mathbf{Q} \tag{26}$$

$$\mathbf{K}^* = \mathbf{P}_{t|t}\mathbf{F}_t^{*\prime}\mathbf{f}_{t+1|t}^{*-1} \tag{27}$$

$$\beta_{t|t,\beta_{t+1}} = \beta_{t|t} + \mathbf{K}^*(\beta_{t+1} - \mathbf{F}\beta_{t|t}) \tag{28}$$

$$\mathbf{P}_{t|t,\beta_{t+1}} = \mathbf{P}_{t|t} - \mathbf{K}\mathbf{F}\mathbf{P}_{t|t} \tag{29}$$

In this case, recursion begins at the time $t = T$, and proceeds backwards to $t = 1$. The first two equations are used for the calculation of the forecast errors, η^*, and the respective variances, $\mathbf{f}^*$, for the time $t+1$ using the information until the time t. It is worth stressing that $\eta^*_{t+1|t}$ and $\mathbf{f}^*_{t+1|t}$ are calculated for the vector of unobserved variables, unlike the earlier recursion in which $\eta_{t|t-1}$ and $\mathbf{f}_{t|t-1}$ are calculated for the observed

19 A detailed derivation and description of updating equations in the Kalman filter are given in Hamilton (1994).

endogenous variables. After the Kalman gain, $\mathbf{K}^*$ is calculated, the vector of unobserved variables and the adequate variance for the time t are updated again, using the above mentioned indicators for time $t+1$. For such estimated values for $\bar{\beta}_{t|t,\beta_{t+1}}$ and $\bar{\mathbf{P}}_{t|t,\beta_{t+1}}$ numerical simulation can be drawn from the normal distribution $p(\beta_T|\mathbf{Q},\mathbf{R},\mathbf{F},\tilde{\mathbf{Y}}_t,\beta_{t+1},X_t)$. This process continues for every time t until $t=1$.

The described recursions are evaluated for the first step at every iteration of the Gibbs sampler. Eventually, after M iterations for each time-varying parameter in the matrix $\tilde{\beta}_T$ as well as for each of the hyperparameters in the matrices $\mathbf{R}$ and $\mathbf{Q}$ final posterior distributions will be numerically approximated using the numerical values obtained in the last $M-K$ iterations. However, this paper does not aim at estimating parameters, but forecasting. For this reason, in every iteration of the Gibbs sampler for the future periods $T+1,\ldots,T+H$ endogenous variables will be forecasted using the measurement equation. In accordance with the existing literature, it will be assumed that for each iteration of the Gibbs sampler the value of time-varying parameters in the forecasting horizon is equal to the values from time T.[20] Accordingly, for the TVP-BVAR model with an exogenous variable for each m iteration and for each h horizon, the following forecast applies:

$$\bar{Y}_{T+h}^{(m)} = c_T^{(m)} + \mathbf{B}_T^{(m)}\bar{Y}_{T+h-1} + D_T^{(m)}x_{T+h} + u_{T+h}^{(m)} \tag{30}$$

The simulated series of $M-K$ forecasts for endogenous variables will approximate the posterior distribution of interest for $\bar{Y}_{T+h}$. The final central forecast for $\bar{Y}_{T+h}$ is calculated as the mean of all $M-K$ iterations.

9 Appendix 2 Diebold-Mariano test

The Diebold-Mariano test compares two series of forecast errors for a specific horizon h. The errors for each horizon h for model 1 or model 2 can be denoted as $\varepsilon_{t+h|t}^1$ or $\varepsilon_{t+h|t}^2$. The loss functions for such errors in this paper are squared and denoted in the following way: $L(\varepsilon_{t+h|t}^i) = (\varepsilon_{t+h|t}^i)^2$, for $i=1,2$. By testing the following hypotheses, it will be possible to check whether the forecast obtained by model 1 differs significantly from the one obtained by model 2:

$$H_0: E[L(\varepsilon_{t+h|t}^1)] = E[L(\varepsilon_{t+h|t}^2)] \tag{31}$$

$$H_1: E[L(\varepsilon_{t+h|t}^1)] \neq E[L(\varepsilon_{t+h|t}^2)] \tag{32}$$

If we write the difference between the error of model 1 and the error of model 2 for horizon h as $d_{t,h} = L(e_{t+h|t}^1) - L(e_{t+h|t}^2)$, the Diebold-Mariano test-statistics is equal to the following expression:

$$S_h = \frac{\bar{d}_h}{\sqrt{\hat{V}(\bar{d}_h)/T}} \to N(0,1) \text{ with } H_0 \tag{33}$$

where $\hat{V}(\bar{d}_h)$ is the robust variance estimator of the difference in errors by Newey and West (Newey and West, 1987), and $\bar{d}_h$ is the average of the differences of errors for the iterations from 1 do T for horizon h, or $\bar{d}_h = \frac{1}{T}\sum_{t=t_0}^{T} d_{t,h}$. In this case, it is important to use the robust variance because there is a possibility of autocorrelation and heteroscedasticity of forecast errors for horizons $h>1$ (Harvey, 1997).

20 The reasons for such an assumption are explained in Sbordone and Cogley (2008).

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