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Precautionary Saving, Wage Risk, and Cyclical Reallocation

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Precautionary saving, wage risk, and cyclical reallocation*

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Abstract

Workers, along with the risk of unemployment, face the wage risk coming from employment-employment reallocation. I examine the consequences of cyclical wage risk variation within a model incorporating search and matching frictions, incomplete markets, and nominal rigidities. In this economy, endogenous countercyclical wage risk can trigger a significant destabilizing demand feedback driven by a precautionary saving motive. I demonstrate that a dynamic job ladder amplifies the response to productivity shocks through changes in the reallocation rate and cyclical variations in relative wages. I argue that both channels are empirically plausible.

Keywords: income risk; imperfect insurance; on-the-job search.

JEL classification numbers: E21, E24, E32, J64

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1 Introduction

Frictional labor markets have the potential to destabilize the economy due to cyclical income risk. When the labor market is slack, workers face a high probability of job separation and a lower chance of job finding. High uninsured unemployment risk boosts precautionary saving, which depresses aggregate demand and output, and further worsens labor market conditions. However, cyclical two-way labor flows between unemployment and employment are not the only source of labor income risk. About one-half of all hires are made directly from the pool of employed, bypassing unemployment (Fujita, Moscarini and Postel-Vinay, 2020). The employment-employment transitions reflect the gradual reallocation of workers to more productive jobs, typically associated with higher wages. This phenomenon presents an additional source of labor income risk, namely wage risk, which pertains to the part of labor income risk unrelated to unemployment risk. As the rate of employment-employment reallocation is cyclical, it implies that wage risk also fluctuates with the business cycle. While literature extensively explores unemployment risk, the relevance of time-varying wage risk in shaping destabilizing precautionary asset demand is often overlooked, and its impact on aggregate volatility remains less understood.

In this paper, I investigate the implications of uninsured wage risk for time-varying precautionary asset accumulation and its contribution to the propagation of aggregate shocks. To this end, I embed a frictional labor market in a Heterogeneous Agent New Keynesian (HANK) model with nominal rigidities and imperfect insurance against income risk. In my model, workers are risk averse and can accumulate liquid wealth to self-insure against the adverse realization of idiosyncratic income risk stemming from the frictional labor market. The labor market incorporates search and matching (SAM) frictions in a Diamond - Mortensen - Pissarides tradition. To capture the wage risk associated with cyclical reallocation to more productive matches, I assume that workers are also searching on the job. In this setting, workers experience progressive wage increments related to climbing the job ladder, interspersed with abrupt throwbacks to unemployment. This pattern of career advancement with occasional setbacks shapes households' earning dynamics and ultimately determines wage risk. As the pace of employment-employment reallocation depends on the labor market conditions, the wage risk is cyclical, which gives rise to time-varying precautionary asset accumulation.

I show that the on-the-job search affects workers' time-varying precautionary saving motive through two channels, namely, the *reallocation* channel and the *relative wage* channel. The reallocation channel relates the labor market tightness with the probability of transition to a better match. When a negative shock hits the economy, a slack labor market translates into a low pace of reallocation towards more productive matches; workers climb the job ladder slowly, which increases downward wage risk and reduces upward wage risk, causing precautionary savings to rise. The relative wage channel stems from the cyclicality of relative wages in the economy with the on-the-job search. As poaching of a worker can destroy matches, the value of a match depends on the job-to-job transition rate. If workers have some bargaining power, fluctuations in the job-to-job transition rates lead to a change in bargained wage, especially for low-productive matches where the probability of poaching is high. For instance, during downturns, a weak labor market impacts wages positively at the lower end of the productivity distribution because a sluggish job market reduces the likelihood of worker poaching, thus increasing the value of a match. Conversely, at the top of the productivity distribution, this channel is less pronounced, or even absent, because high-productivity matches are not as vulnerable to poaching. Consequently, cyclicality in job-to-job transition rates causes the wage differentials between low- and high-productivity matches to decrease during recessions, which I show analytically. This time-varying difference presents an additional source of wage risk, giving rise to a procyclical precautionary saving motive for high-productivity matches and a countercyclical precautionary saving motive for low-productivity matches.

To demonstrate how these two forces unfold, I consider a limiting case of my model with a zero asset supply, akin to the approach taken by Ravn and Sterk (2017, 2021) and Challe (2020). I construct an equilibrium wherein the only type of workers not liquidity-constrained are those positioned at the upper bound of the productivity distribution. I show analytically that in this economy, the two channels, namely the reallocation channel and relative wage channel, operate in opposite directions. On the one hand, because the precautionary demand for assets originates only from workers at the top of the job ladder, the procyclical wage difference between low- and high-productivity jobs contributes to stabilizing the economy. Intuitively, during recessions, workers are less concerned about descending to the bottom of the job ladder because it implies a smaller wage decline compared to regular times. The reduction in downward wage risk leads to a decrease in precautionary savings, thereby stabilizing the economy. On the other hand, a time-varying job-to-job reallocation rate generates a countercyclical precautionary saving motive, acting as a destabilizing force. In bad times, destabilization occurs because if workers separate, it becomes more difficult to ascend to the top of the job ladder due to sluggish job-to-job reallocation. Consequently, workers fear finding themselves stuck in low-productivity matches, which increases downward wage risk and prompts precautionary saving. However, analysis of the zero-liquidity equilibrium is restrictive as it abstracts from considering the effect of cyclical wage risk on the savings demand of low-productive and unemployed workers.

To quantify the effect of wage risk in a more realistic setting, I relax the assumption of zero liquidity and evaluate the time-varying precautionary saving motive in an economy with non-degenerate wealth distribution. To this end, I calibrate my quantitative model to match the key moments of the US data, including liquidity, labor market flows, and residual wage dispersion. As a result, the model can replicate various salient data features, such as the negative skewness and high kurtosis of the income growth distribution, which is a natural outcome of job-to-job movements.

The novelty of this paper lies in demonstrating the significant destabilizing role of demand feedback stemming from the cyclical job ladder. Quantitatively, I show that aggregate productivity shocks undergo substantial amplification through both the reallocation and relative wage channels. These two forces increase unemployment and consumption responses by up to one-third compared to a counterfactual scenario where these channels are not operative. Furthermore, I demonstrate that in the quantitative model, the *composition* effect becomes another source of amplification. The composition effect arises in models with cross-section heterogeneity because labor market frictions not only alter perceived uncertainty but also induce actual shifts in the composition of the labor force. While in a conventional HANK&SAM economy without on-the-job search compositional changes typically stabilize the economy, I show that a combination of the job ladder and tight borrowing constraints can reverse this standard result.

Related literature. This paper contributes to the burgeoning literature on the role of income risk in the HANK economies. Werning (2015) demonstrates that income risk cyclicality generally shapes the aggregate consumption response to exogenous shocks. Challe and Ragot (2016), Ravn and Sterk (2017) show that endogenous countercyclical unemployment risk coming from labor market imperfections leads to destabilizing demand feedback due to a precautionary saving motive. Ravn and Sterk (2021) derive this result theoretically within a tractable zero-liquidity setup: a combination of debt limits and zero net asset supply ensures degenerate wealth distribution. Challe (2020) and Bonciani and Oh (2021) show that in a zero-liquidity economy, the appropriate monetary policy response almost entirely offsets the destabilizing impact of endogenous unemployment risk. Broer et al. (2021), and Jung (2023) examine the role of endogenous job destruction in precautionary saving feedback. I contribute to the literature by analyzing the role of cyclical wage risk (in addition to unemployment risk) stemming from cyclical job reallocation. In my framework, the zero-liquidity economy can arise as a limiting case, and the effect of wage risk on aggregate demand is generally ambiguous.

Several studies depart from the assumption of zero liquidity and explore the effect of income risk in the economy featuring non-degenerate wealth distribution. Cho (2023) documents that in an economy with wealth heterogeneity and search and matching frictions, income risk plays a very limited role because of the strong composition effect. This finding contrasts with the effect of cyclical income risk in the economies where the composition effect is absent either because of limited heterogeneity (Challe et al., 2017) or because changes in income risk are exogenous (Bayer et al., 2019). Graves (2020) shows that a two-asset economy features strong precautionary saving demand feedback even though the economy experiences a change in unemployment-employment composition. In my model households can save only in unique liquid assets, but the shock may exhibit strong amplification from the precautionary channel because the job ladder can revert the composition effect.

The frictional labor market with on-the-job search in my model relies on the assumptions of random search and bilateral bargaining in the spirit of Pissarides, as opposed to the wage posting models. For the wage determination, I use the result of Gottfries (2018), that for the economies with heterogeneous employer-worker matches and infrequent wage renegotiation, equilibrium wage maximizes the Nash product. For the empirical test of the model implications, I employ the methodologies of Faberman, Justiniano et al. (2015); Moscarini and Postel-Vinay (2017*b*); Karahan et al. (2017), which explore the performance of various labor market flows as predictors of individual wage growth.

Finally, several recent works incorporate job ladders in a general equilibrium framework with risk-averse households. Moscarini and Postel-Vinay (2017*a*) show that cyclical movement in employment allocation induced by job-to-job activities can affect inflation dynamics. Faccini and Melosi (2021) use this insight to explain the missing inflation puzzle. Further, Alves (2022) demonstrates that cyclical job reallocation following financial shock can account for the missing disinflation in the quantitative HANK & SaM framework and Birinci et al. (2022) analyzes this environment from the normative point of view. Similarly to these papers, I incorporate a stochastic job ladder in the general equilibrium model with risk-averse households but concentrate on aggregate demand rather than inflation dynamics.

2 Wage adjustment in the model with on-the-job search.

I start by considering a simple economy with a frictional labor market and risk-neutral workers. The purpose of this section is to show how the introduction of on-the-job search changes wages in the stationary equilibrium and the properties of cyclical wage adjustment. Specifically, I demonstrate that the possibility of job-to-job reallocation reduces wages for low-productive matches and makes wages relatively more volatile at the top of productivity distribution. This wage dynamics becomes a source of additional countercyclical income risk in the economy with risk-averse households, which I will develop in Section 3.

I rely on the framework of Gottfries (2018) featuring random search and non-cooperative wage bargaining. Wage determination unfolds through the bargaining game of Binmore, Rubinstein and Wolinsky (1986), wherein a firm and a worker alternate in proposing wage contracts. The turnover rate depends on contracted wages and contracted wages are infrequently renegotiated, which allows for overcoming the problem of a non-convex bargaining set (Shimer, 2006) that might be an issue in economies with on-the-job search (Pissarides, 1994; Mortensen, 2003).

2.1 Environment

The economy consists of workers and firms.

Workers are ex-ante identical infinitely-living risk-neutral agents. Each worker can be in one of two states: employed or unemployed. When unemployed, workers are looking for a job. During the search, unemployed workers receive a flow of real return b, which can be interpreted as home production or unemployment insurance benefits. When employed, workers are matched with firms and earn match-specific wages $w(x)$, where x is the idiosyncratic productivity of a match. Besides, employed workers always search for a better job, but with lower intensity than unemployed.

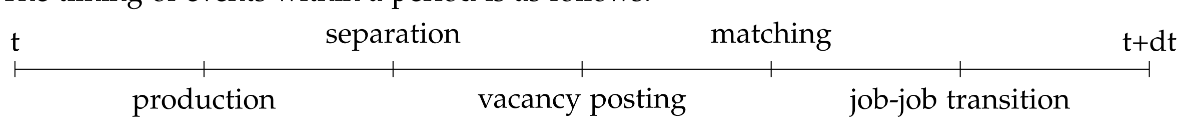
Ex-ante identical risk-neutral firms are on the other side of the labor market. Each firm has one job, which can be either filled or vacant. Firms with a vacant job search for an employee by posting vacancies at a per-period real cost of c. Firms with filled jobs produce homogeneous consumption goods with linear technology using only labor as input. If a firm neither produces nor searches, its job is destroyed.

The productivity of each firm consists of idiosyncratic component x, which is match-specific, and aggregate component z, which is common for all firms in the economy. The worker-firm separation occurs for two reasons. First, an idiosyncratic adverse shock hits a firm with Poisson intensity λ and destroys the match. In this scenario, a worker engaged in the match becomes unemployed. The second possibility for job destruction is poaching. Separation occurs if the worker can form a better match with an outside firm offering a job. In this case, workers change their jobs bypassing the unemployment state, in contrast to the separation after an adverse shock. However, from the firm's point of view, separation induced by shock and poaching are equivalent.

I denote the total population of workers in the economy as L. The corresponding number of unemployed workers and vacant jobs are uL and vL, respectively. The efficient number of job seekers is $uL + s(1-u)L$, where $1 \geq s \geq 0$ refers to the search efficiency of employed workers. To ensure analytical tractability, I assume $s = 1$ throughout this section.

The number of job matches is given by homogeneous of degree one matching function $m(.,.)$, such that $mL = m(vL, (u + s(1-u))L)$. Defining labor market tightness as $\theta = \frac{v}{u+s(1-u)}$, the rate at which workers arrive to vacancies is $\frac{mL}{vL} = m(1, \frac{1}{\theta}) \equiv q(\theta)$. Workers' corresponding job arrival rate is $\theta q(\theta)$.

The timing of events within a period is as follows:



1. Firms produce consumption good according to their productivity px and pay to workers wage $w(x)$. Unemployed households enjoy benefits b.

2. Adverse separation shocks hit the existing matches with rate λ.

3. Unemployed and employed workers search for jobs, firms post vacancies, and random matches

occur.
4. When a worker meets with a firm, they draw random productivity value from the known distribution $G(x)$, with $G(\overline{x}) = 0$ and $G(\underline{x}) = 1$. Given the productivity of the match x, the firm and potential employee bargain for the wage.
5. Workers decide whether to accept the job with wage $w(x)$.

2.2 Equilibrium wage

Wage contracts do not last forever, as in Gottfries (2018). Instead, wages are infrequently renegotiated with an exogenous intensity independent of the offer arrival rate. Throughout the paper, I assume that the frequency of renegotiation approaches infinity.

When a firm and a worker are matched, they negotiate wages via the strategic bargaining game of Binmore, Rubinstein and Wolinsky (1986). Players take turns making offers in a bargaining game, with the firm initiating the first offer. Following each offer, the responder decides whether to accept or reject it. Accepted offers result in the agreed-upon wage payoffs for both parties, while rejected offers carry a probability of leading to a breakdown in the bargaining process. After the worker extends an offer, the probability of no breakdown is $(1 - dt)^{\phi}$, while it is $(1 - dt)^{1-\phi}$ following an offer from the firm. In the event of a breakdown, the worker transits to unemployment, and the firm exits without any additional value.

The key insight of Gottfries (2018) is that in an on-the-job search model with bargaining, infrequent renegotiation of wages, and match-specific productivities, the equilibrium wage maximizes the Nash product even though the bargaining set might be non-convex. I use this result to derive an equilibrium wage in my economy. Denoting worker surplus as S^w and firm surplus as S^f, the bargaining game has a unique solution, such that $w = \arg\max_{w} (S^w)^{\phi}(S^f)^{1-\phi}$. The following proposition establishes wages as a result of the bargaining game

Proposition 1. *The wage resulting from the bargaining game is given by*

$$w(x) = (1-\phi)b + \phi z(x + \theta c\Theta) - \underbrace{\phi f^{ee} \int_{x}^{\overline{x}} J(s)dG(s)}_{\Delta(x)} \tag{1}$$

where $J(x)$ is a value of a firm with productivity x, $f^{ee} = s\theta q(\theta)$ and $\xi = \frac{\int_{\underline{x}}^{\overline{x}} J(s)dG(s)}{\int_{\underline{x}}^{\overline{x}} F(s)J(s)dG(s)}$. See proof in Appendix A

Consider the first two terms in Equation (1). Wage is growing in benefit b since high unemployment benefit increases the outside value of the worker. Besides, workers receive part of the output zx, and the firm's opportunity costs $z\theta c$, proportional to the bargaining power ϕ. These two terms constitute a standard wage expression in models *without* on-the-job search as, for example, in Mortensen and Pissarides (1994) (without on-the-job search $\Theta = 1$).

Additional term $\Delta(x)$ in Equation (1) differs wage expression in this economy from the setups that either abstract from job-job reallocations or incorporate it in the more stylized form as in Pissarides (1994). Term $\Delta(x)$ corresponds to the compensation for the expected poaching of a worker. When job-to-job transitions occur, workers climb the job ladder and experience an increase in utility (value) stemming from higher wages, contrasting with firms, for whom the loss of an employee (due to poaching) implies job destruction. Thus, at a given wage, the introduction of on-the-job search impacts the value of workers and firms in opposite directions: it increases the value of workers while decreasing the value of firms. Since the identity $\phi S_f = (1-\phi)S_w$ must hold, the new equilibrium wage mechanically decreases, as it should account for the expected job-to-job transitions of workers. Generally, it implies that the negotiated wage is lower than in the baseline result of Mortensen and Pissarides (1994).

From Equation (1), it is clear that $\Delta(x)$ is necessarily non-negative and increasing in labor-market tightness θ. This property is not surprising since high labor-market tightness increases the job arrival rate and boosts the probability of poaching. Another essential feature of $\Delta(x)$ is that it is decreasing in x, with $\Delta(\overline{x}) = 0$. Poaching is more likely when a match is low-productive, while poaching is impossible at the upper bound of productivity distribution $\overline{x}$.

Equation (1) provides a wage expression, albeit not in closed form, as the firm value $J(x)$ is an endogenous object. Additionally, the exact form of $\Delta(x)$ depends on the productivity distribution $G(x)$. Fortunately, for certain distributions $G(x)$, the term $\Delta(x)$ can be expressed analytically. Proposition 1 establishes equilibrium wage in closed form for the uniform productivity distribution.

Proposition 2. *Assume that $G(x)$ is a continuous uniform distribution with support $x \in [\overline{x}, \underline{x}]$. In this economy, the wage schedule is given by*

$$w(x) = (1-\phi)b + \phi z(x + \theta c \Theta) - \underbrace{\phi z(\bar{x} - x) - \bar{x}z\left[\frac{C(x)^{\phi} - C(\bar{x})^{\phi}}{C(\underline{x})^{\phi-1}}\right]}_{\Delta(x)} \tag{2}$$

where $C(x) = \frac{r+\lambda}{\theta q(\theta)} + 1 - \frac{x}{\bar{x}}$. See proof in Appendix A

To illustrate the effect of on-job search on wage function, I numerically find equilibrium in the model[1] using calibration that matches conventional moments in the data.[2] Figure 1, plots equilibrium wage as a function of productivity match x.

At the lower support of the productivity distribution equilibrium wage in the model with on-the-job search is reduced compared to the models where employees do not search. When id-

[1]Equilibrium to this economy is defined by wage $w(x)$, labor-market tightness θ, and firm distribution $F(x)$ which are consistent with the wage equation, and the job creation condition distribution provided by Equations 2, **??**, and **??**.

[2]The model is in monthly rates. The separation rate is $\lambda = 0.034$, search intensity ensures that the job-finding rate $\theta q(\theta) = 0.45$. Workers' bargaining power $\phi = 0.5$. The Hosios efficiency condition is satisfied. The vacancy posting costs c are those that match the labor-market tightness $\theta = 1$. Productivity $G(x) \sim U[0.9, 1]$.

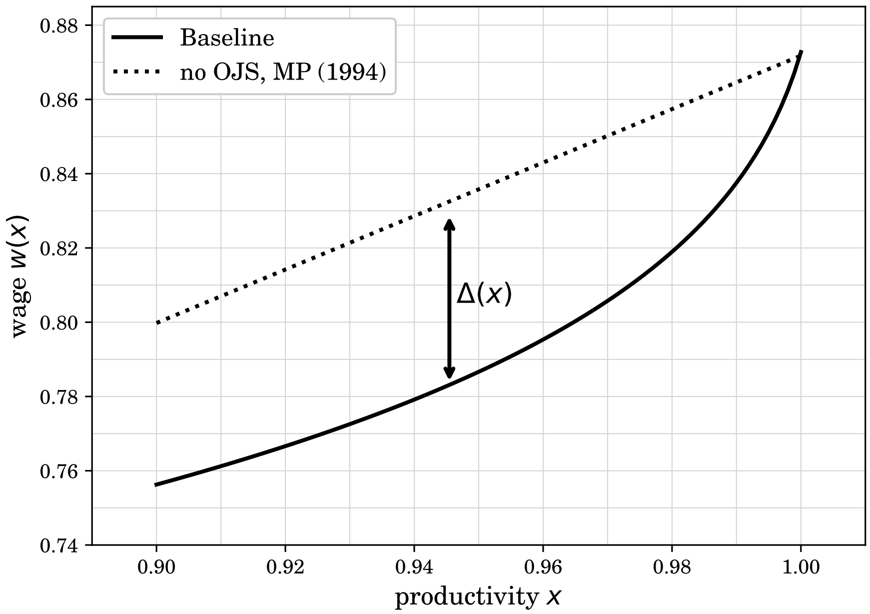


Figure 1: Wage schedule in the model with on-job search

The solid line represents wage as a function of idiosyncratic productivity x in the model with on-the-job search laid out in this section. The dotted line is the wage equation as in Mortensen and Pissarides (1994) where the on-the-job search is absent.

iosyncratic productivity is small, the probability of poaching is high, translating into a larger compensation term $\Delta(x)$. It means that, as a consequence, workers and firms agree to set wages at a lower level through negotiation. When idiosyncratic productivity is high, there is a lower chance that a firm with random productivity lures away workers, therefore term $\Delta(x)$ shrinks and the negotiated wage is higher. The resulting wage schedule is much steeper compared to the case with no on-the-job search, implying that the transition from low-productive to high-productive matches is generally associated with larger increases in wage than in Mortensen and Pissarides (1994).

2.3 Cyclical wage adjustment

Next, I examine how the wage adjusts at different parts of the productivity distribution following a change in aggregate productivity. To this end, I consider a permanent decrease in productivity component z that is common for all matches in the economy.

A negative productivity shock has a conventional effect on the economy. Low productivity decreases the number of new vacancies, the labor market tightness, and ultimately job-finding and job-job transition rates. With lower productivity and a weaker labor market, workers renegotiate wages. Formally, a change in wages could be expressed as:

$$\frac{dw(x)}{dz} = \phi(x + \theta c\Theta) + \phi z c\Theta \frac{d\theta}{dz} - d\frac{\phi f^{ee} \int_x^{\overline{x}} J(s)dG(s)}{dz} \tag{3}$$

The first and the second terms in Equation (3) relate the change in wage with the direct effect of productivity (direct channel) and with the change in labor market tightness (opportunity costs channel). The main object of interest is the third term in Equation (3), which enters the equation with the negative sign. When the labor market is slack, the contact rate is low, and the probability of poaching declines, which shrinks the $\Delta(x)$ term. Graphically, Figure 2 illustrates the overall impact of lower productivity on equilibrium wage.

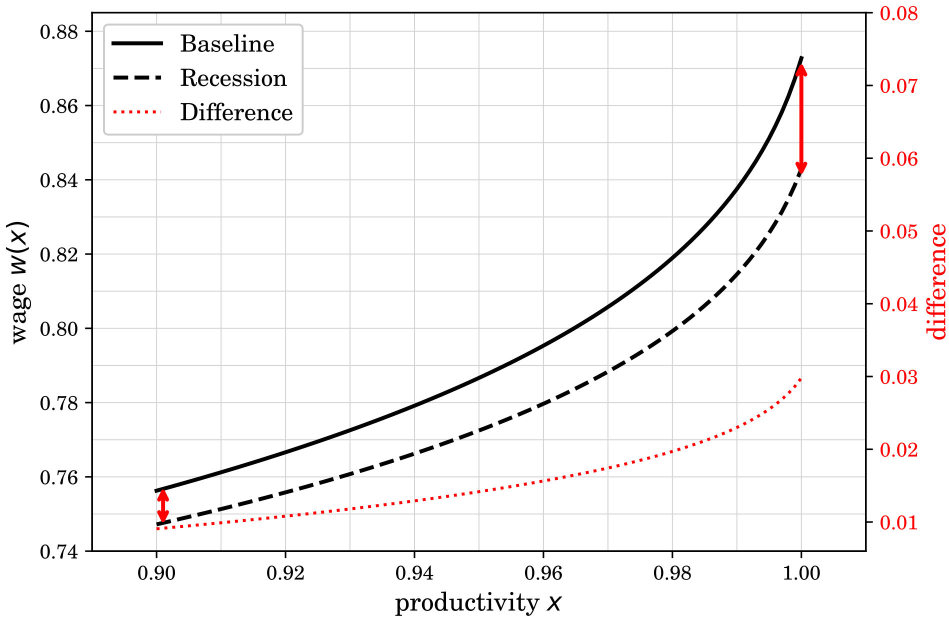


Figure 2: Wage schedule in the model with on-job search

Black curves plot the steady-state wage functions in the model with on-the-job search before and after a permanent decrease in aggregate productivity z. The red line shows the difference between the two wage functions

As the dashed line indicates, a decline in aggregate productivity shifts wages for every idiosyncratic productivity level x. The change in wages originates from the (i) direct, (ii) opportunity costs, and (iii) poaching channels. However, the shift is not parallel: at the top of the productivity distribution, bargained wage drops dramatically, while for low x wages experience only a moderate decline. This asymmetry comes from the poaching channel; when the labor market is slack, the $\Delta(x)$ term declines, which has a disproportionately strong positive impact on wages at the bottom of the productivity distribution. As a result, a fall in productivity makes the wage differentials between more productive and less productive jobs smaller.

2.4 Discussion

This section highlights the role of on-job search in cyclical wage adjustment. Workers extract part of the total surplus because of their bargaining power. The size of the total surplus depends on the contact rate, and ultimately, the probability of poaching. When aggregate conditions change, wages also change through conventional direct and opportunity cost channels as well as through the changes in poaching probability. The latter impacts wages at the bottom of the productivity distribution, as workers in low-productivity matches are subject to poaching.

The central result of this section is that the wage differential between high-productivity and low-productivity matches decreases during recessions. This wage characteristic becomes crucial in Section 3, where I deviate from the assumption of linear utility and investigate the role of precautionary saving. When the labor market is weak, workers in low-productivity matches not only face a low probability of reallocating to a more productive job but also experience a smaller wage increase with each reallocation due to smaller wage differentials across different positions on the job ladder, compared to normal times.

Importantly, my model differs from the classical framework of Pissarides (1994), which assumes that new matches are always at the upper bound of the productivity distribution. As a result, workers reallocating to a new employer always transit to the very top of the job ladder, the turnover rate doesn't depend on the bargained wage and the poaching channel is absent.

Finally, the main result on wage adjustment is robust to the incorporation of stochastic idiosyncratic productivity and endogenous destruction in the spirit of Mortensen and Pissarides (1994); Pissarides (1994). As I show in Appendix A, both extensions do not change the wage equation. Moreover, the model can be extended to include reallocation shocks or search costs, with the same qualitative results.

3 Quantitative model

In this section, I integrate the key theoretical mechanism from Section 2 into a conventional HANK & SAM model, in the spirit of Ravn and Sterk (2017, 2021) and Challe (2020). Like before, there exists a frictional labor market. However, households are now assumed to be risk-averse, responding to changes in income risk through precautionary saving in the absence of other means of insurance. Additionally, the model includes imperfections in the goods market, which, together with financial and labor market frictions, give rise to powerful amplification mechanisms in general equilibrium.

3.1 The model

Households. The economy is populated by two types of households: workers and firm owners. There is a measure-one continuum of ex-ante identical risk-averse workers participating in the labor market and a measure Ω of risk-neutral firm owners that run all firms in the economy, collect profits, and supply no labor.

Workers can be employed or unemployed. Unemployed workers earn exogenous home production income b_t and search for a job. Employed workers supply one unit of labor inelastically and can be in a match with a firm with one of the productivities x_1, x_2, ... x_n with corresponding wages $w_t(x_{it})$. Besides, employed workers always search for a better job with search intensity s.

Workers maximize lifetime value $W(.)$ by choosing a stream of consumption c_t subject to a budget constraint:

$$\begin{aligned} W(a_{i0}, x_{i0}) &= \max_{\{c_{it}\}_{t\geq 0}} E_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt \\ \dot{a}_{it} &= r_t^r \cdot a_{it} + w_t(x_{it}) - c_{it} \\ x_{it} &\in \{x_1, x_2, ... x_n, 0\} \end{aligned} \tag{4}$$

where $u(.)$ is a utility function with constant relative risk aversion σ; workers discount the future at rate ρ and save in a liquid asset a_t at a riskless real rate of return r_t^r. Here, I use w_t to denote generic labor income coming from either the labor market or home production, such that $w_t(x_t) = b_t$ if $x_t = 0$.

Aside from the budget constraint, workers face exogenous debt limit $\underline{a}$:

$$a_t \geq \underline{a}(x_t) \tag{5}$$

I follow Ravn and Sterk (2021) by assuming that the tightness of the borrowing constraint depends on the labor income. For unemployed workers, the debt limit is zero, such that $\underline{a}(0) = 0$.

Firms. The production structure of the economy incorporates three vertically integrated sectors: labor intermediaries, retailers, and final goods producers. The labor intermediaries hire workers from the frictional labor market and produce labor services, which they sell in the competitive market. In the middle of the supply chain, monopolistic retailers buy labor services and produce specialized intermediate goods, which then are aggregated by final good producers. Besides, monopolistic firms face nominal rigidities in the form of price adjustment costs.

As in Section 2, labor intermediary firms can either have filled jobs or vacant jobs. Firms with filled jobs sell homogeneous labor services with linear production technology and sell it at real price p^I. Separation of a match can occur for two reasons: an exogenous separation shock λ, and poaching that occurs with endogenous probability $f_t^{ee}(1 - G(x))$. The latter depends on the relative search intensity of employed workers, labor market tightness θ, and productivity distribution $G(x)$. Consequently, an effective discount rate denoted as $M^F_{x,t}$ is $r_t + \lambda + f^{ee}_t(1 - G(x))$. The value of a firm with filled job J can be written recursively:

$$J_{x,t} = \underbrace{\Big(zp^I x - w_{x,t}\Big)}_{\text{instantaneous profit}} dt + \underbrace{\Big(1 - M^F_{x,t} dt\Big) J_{x,t+dt}}_{\text{discounted future payoff}} \tag{6}$$

Firms with vacant jobs do not produce and post vacancies at the per-period cost of c. Firms with vacant jobs have value V_t defined as

$$V_t = \underbrace{-\,cdt}_{\text{posting costs}} + \underbrace{\Big(q(\theta) - r_t\Big) dt \int_{\underline{x}}^{\overline{x}} F(s) J_t(s) dG(s)}_{\text{expected return}} \tag{7}$$

The free-entry condition pins down the value of firms with vacant jobs such that $V_t = 0$ at every t.

As in the analytical model, the wage resulting from the negotiation process between workers and firms maximizes Nash product:

$$w^N_{x,t} = \arg\max_w \Big(W_{x,t} - W_{0,t}\Big)^{\phi} \Big(J_{x,t} - V_t\Big)^{1-\phi} \qquad x_t \in \{x_1,\, x_2,\, \dots x_n\} \tag{8}$$

Besides, I assume some degree wage inertia η^w, such that the equilibrium wage is a weighted average of w^N_t and the steady-state wage w_{ss}:

$$w_{x,t} = (w_{ss})^{\eta^w} (w^N_{x,t})^{(1-\eta^w)} \tag{9}$$

The retail sector is a middle production layer of the economy. Retailer firms buy labor services from the labor intermediaries for a competitive price p^I_t. Then, they produce differentiated goods (each firm produces its own good denoted by subscript $j \in [0,1]$) with linear technology, using only labor, and sell it for price $P_{j,t}$. Retailers have monopolistic power and face nominal rigidities in price adjustments. Besides, all retailer firms share a common term for aggregate productivity A_t. The firm's per-period profit is:

$$\Pi(P_{j,t}) = P_{j,t} \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} Y_t - \frac{p^I_t}{A_t} \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} Y_t \tag{10}$$

and the maximization problem is:

$$\max_{\{P_t\}_{t \geq 0}} \int_0^{\infty} e^{-\int_0^t i(s)ds} \left\{ \Pi(P_t) - \Theta^A_t \left(\frac{P_{j,t}}{P_t}\right) \right\} \tag{11}$$

subject to the quadratic price adjustment costs:

$$\Theta_t^A\left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right) = \frac{\chi}{2}\left(\frac{\dot{P}_{j,t}}{P_{j,t}}\right)^2 Y_t P_t \tag{12}$$

where $\chi > 0$ stands for the degree of nominal rigidities in price adjustment. Demand for retailer goods in Equation (10) comes from final good producers that buy differentiated goods Y_j and aggregate them into consumption goods $Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$.

Central Bank and Government. The Central Bank sets the nominal interest rate in line with the Taylor rule.

$$i_t = r^{ss} + \phi_\pi \pi_t \tag{13}$$

The government runs public debt D by issuing one-period risk-less bonds. To finance debt costs the government imposes a lump-sum tax on firm owners. The government budget constraint is

$$\dot{D}_t = r_t \cdot D_t - \tau_t \tag{14}$$

where τ_t is the total amount of taxes levied by the government. I assume that the bonds issued by the government and those issued by the households are perfect substitutes and traded on the same market.

3.2 Equilibrium and solution

Equilibrium in this economy consists of sequences of prices, household policy rules, firm policy rules, job-market transition rates, inflation, taxes, and the distribution of households such that, given the initial distribution:

1. Given aggregate variables, households' and firms' policy rules maximize their corresponding objective functions.
2. Taxes ensure that the government budget constraint holds.
3. The free-entry condition holds.
4. Distribution is consistent with policy rules and aggregate variables.
5. Goods, labor, and asset markets clear. The corresponding market-clearing conditions are:

$$\begin{aligned} Y_t &= C_t + \Theta_t^a + cv_t \\ N_t &= \sum_x \int_a x\Phi(a,x,t)da \\ D_t &= \sum_x \int_a a\Phi(a,x,t)da \end{aligned} \tag{15}$$

To derive policy functions, I reformulate household and firm problems as a system of partial differential equations known as Hamilton-Jacobi-Bellman (HJB) equations. Following Achdou et al. (2022), I discretize the system using the upwind scheme method and solve it using the finite-difference method for derivative approximation. I also formulate a system of corresponding Kolmogorov Forward (KF) equations to track the joint evolution of income and wealth distribution, given households' optimal policy.

3.3 Calibration

Calibration. I calibrate the model to match key features of the U.S. data. The model is in monthly rates. I chose the separation rate λ equals 0.034, which corresponds to the monthly separation rate according to Shimer (2005). I set the search elasticity of matching η and worker's bargaining power ϕ to 0.5, as in Mortensen and Pissarides (1994). To calibrate the productivity density function $g(x)$ and subsequently demonstrate its implications (see Section 3.3), I adopt the approach outlined by Alves (2022), which involves matching moments of residual wage dispersion. As the empirical target, I've chosen the logarithmic differences between the 90th and 50th percentiles (0.55) and between the 50th and 10th percentiles (0.60) of the residual wage distribution. (Autor, Katz and Kearney, 2008)

Parameter	**Target**	**Targeted Moment**
Labor Market Flows		
λ	separation rate	0.034
m	job-finding rate	0.45
s	job-job transition rate	0.025
c	labor market tightness	1
Income Process		
g	wage dispersion p50/p10, p90/p50	0.60, 0.55
b	unemployed consumption decline	15%
New-Keynesian Part		
ϵ	aggregate markup	20%
ξ	frequency of price adjustment	12 months
ϕ_π	Taylor rule coefficient	1.2
Liquidity		
ρ	interest rate	2%
$\underline{a}$	fraction with negative assets	0.15
D	debt-output ratio	1.0
Microfounded		
σ	relative risk aversion	2
η	search elasticity of matching	0.5
ϕ	worker's bargaining power	0.5

Table 1: Calibration

I calibrate parameters of matching efficiency, search intensity of employed workers, and cost of vacancy posting to match the job-finding rate, job-to-job transition rate, and labor market tightness. Matching efficiency ensures that unemployed workers find a job with a monthly rate of 0.45. The relative search intensity of employed workers s is calibrated to fit the average job-to-job transition rate of 0.025 as documented by Fujita, Moscarini and Postel-Vinay (2020). Finally, the vacancy posting cost c guarantees that the labor-market tightness θ is 1 (Krusell, Mukoyama and Şahin, 2010).

The relative risk aversion is set to 2 for my baseline calibration. The time discounting parameter ρ matches the interest rate of 2% (in annual terms). I choose the size of home production b to match a 15% average consumption decline associated with the transition to unemployment. Wage persistence η^w is calibrated to 0.9. The elasticity of substitution for the inputs ϵ of the final good producers is set to match an average markup of 20%. I picked the price adjustment costs ξ to match the average price adjustment frequency of 12 months. Finally, the parameter of the Taylor rule ϕ_π is 1.2.

3.4 Model performance

The accuracy of model predictions critically depends on the properly established incentives of households. In particular, to adequately capture precautionary saving behavior, it is essential to set up a plausible amount of income risk in the model and, more broadly, correctly specify the underlying income process.

In the previous section, I explained how I chose parameters of productivity distribution to match the data moments on residual wage dispersion and labor market flows. Now I turn to the model's performance under this parameter choice. Specifically, I am interested in the ability of the model to replicate untargeted moments of labor earnings growth.

For the empirical counterpart, I select the high-order moments of yearly labor earnings growth distribution reported by Guvenen et al. (2021). To ensure comparability, I construct the yearly labor income w^y_{it} as a sum of labor earnings over 12 months $w^y_{it} = \int_1^{12} w_{ih} dh$, with labor income growth defined as $\log w^y_{it+1} - \log w^y_{it}$. To obtain the income growth distribution I simulate a panel of 100,000 households in the stationary equilibrium. Table 2 reports the result.

Moment	**Data**	**Model**
$Var[\Delta w^y]$	0.26	0.11
$Skew[\Delta w^y]$	-1.07	-0.41
$Kurt[\Delta w^y]$	14.93	9.36
$Share \geq 2\sigma$	0.066	0.069
$Share \geq 3\sigma$	0.024	0.017

Table 2: Moments of income growth distribution

The model effectively captures two key characteristics of income growth distribution: negative skewness and positive excess kurtosis. These characteristics arise from the interplay of two essential dynamics: the gradual advancements in labor earnings as workers reallocate into more productive matches, and the sudden setbacks caused by separation shocks, which effectively thrust individuals back to the lower rungs of the job ladder. Several works demonstrate that this disproportion between progressive increments and sudden reversals of labor earnings can generate negative skewness (Hubmer, 2018; Alves, 2022). Furthermore, this theoretical mechanism finds empirical support (Guvenen et al., 2021).

In the data, the labor earnings growth distribution demonstrates a larger skewness compared to what the model produces. Empirically, around half of the skewness in earnings originates from hours skewness, which the model abstracts from. Additionally, the model somewhat underperforms in terms of variance. This is because the model incorporates wage risk associated only with the job ladder, which accounts for only part of the labor income variance observed in the data. Notably, the model effectively replicates the high empirical density at the tails of the earnings growth distribution, resulting in high kurtosis.

Next, I investigate the model's capacity to replicate stylized features of the business cycle. To accomplish this, I explore the MIT shock and derive the model's solution in terms of the entire history of exogenous shocks, as proposed by Boppart, Krusell and Mitman (2018). Having obtained the solution, I conduct a simulation of the model and then calculate moments of the main labor market flows. Results are presented in Table 3.

Model	y	v	f	u
Relative standard deviation	1.00	10.88	11.92	8.30
Correlation with output	1.00	0.91	0.95	-0.79
Correlation with unemployment	-0.79	-0.65	-0.71	1.00
Data	y	v	f	u
Relative standard deviation	1.00	10.06	11.01	9.83
Correlation with output	1.00	0.59	0.86	-0.79
Correlation with unemployment	-0.79	-0.67	-0.93	1.00

Table 3: Labor market moments

The model reasonably captures stylized features of the US labor market. It generates highly procyclical job-finding rates and vacancy postings, along with countercyclical fluctuations in unemployment, which align with the data. The model matches the volatility of job-finding rates and vacancy postings quite well but somewhat underperforms in terms of the volatility of unemployment. Overall, the model performs sufficiently well in replicating key aspects of the business cycle dynamics in the labor market.

4 Results

4.1 Limiting case: zero-liquidity economy

Before turning to the quantitative results based on the calibration proposed in Section 3, I take a detour to examine a special case of my model. I assume only two productivity states: high x^h and low x^l, with respective wages w_t^h and w_t^l. Besides, I assume that unemployed workers and those workers in low-productivity matches have limited access to the asset market and cannot issue private bonds. Additionally, I consider a scenario where the government does not provide liquidity.[3]

It is straightforward to show that there exists an equilibrium in this economy, where tight borrowing constraints ensure that unemployed workers and those in the low-productivity state are hand-to-mouth workers. For these workers, the Euler equation holds with inequality: they would borrow against future income but are unable to do so. Conversely, workers in the high-productivity state are not hand-to-mouth, and the Euler equation holds with equality for them. As there is zero liquidity in this economy, the interest rate adjusts so that, in equilibrium, workers in the high-productivity state neither save nor borrow. Thus, borrowing constraints and the zero-liquidity assumption maintain a no-trade equilibrium and eliminate cross-sectional wealth heterogeneity.

To highlight the significance of precautionary saving in the economy's adjustment following a productivity shock, I adopt a conventional approach in the HANK & SAM literature. I examine the Euler equation of workers in high-productivity matches, where both precautionary saving and consumption smoothing motives remain operative:

$$\frac{r_t^r - \rho}{\sigma} = \overbrace{\frac{\dot{w}_t^h}{w_t^h}}^{\substack{\text{intertemporal}\\ \text{substitution}}} - \underbrace{\lambda(1-f_t)\frac{w_t^h - b_t}{w_t^h}}_{\text{unemployment precautionary}} - \underbrace{\lambda\frac{f_t^l}{2}\left(\left(1-\frac{f_t^{ee}}{2}\right)+\left(1-\frac{f_t}{2}\right)\right)\frac{w_t^h - w_t^l}{w_t^h}}_{\text{wage risk precautionary}} \tag{16}$$

where f_t is probability to find *any* job during dt and f_t^l is probability to find a low-productivity job.

Equation (16) is the continuous-time counterpart of the equilibrium condition found in Challe (2020), as well as Ravn and Sterk (2021), extended to accommodate two productivity states. It demonstrates that the change in desired savings and, consequently, the interest rate response in this economy is driven by two fundamental effects: consumption smoothing and precautionary saving.

The consumption smoothing motive arises from the worker's aversion to intertemporal substitu-

[3] To obtain analytical results, I also adjust the time convention to align with conventional HANK&SAM models with zero liquidity. See Appendix B for details.

tion, with the first term on the right-hand side of Equation (16) capturing this effect. The other two terms governing household optimal policy are broadly associated with precautionary saving.[4] The second term reflects the impact of unemployment risk on precautionary saving and the associated cuts in consumption level from w_t^h to b_t. The last term of Equation (16) introduces a precautionary term stemming from the wage risk, which, as far as I am aware, has not been previously discussed in the literature. It demonstrates that the job ladder propagates aggregate shocks through the change in employment-employment transition rates f^{ee} and through in relative wages $w_t^h - w_t^l$. Note, when $f_t^l = 0$, Equation (16) nests a conventional zero-liquidity HANK&SAM result as a special case.

Next, let's consider the local dynamics of the zero-liquidity economy around the steady state commonly examined in the New Keynesian literature.[5] To denote percentage deviations from the steady state, I use a tilde, while absolute deviations are indicated by a hat sign. The first-order approximation of Equation (16) can be expressed as:

$$\hat{r}_t = \sigma \tilde{w}_t^h + \psi_1 \hat{f}_t + \underbrace{\psi_2 (1 - \frac{w^l}{w^h}) \hat{f}_t^{ee}}_{\text{reallocation}} + \underbrace{\psi_3 \frac{w^l}{w^h} (\tilde{w}_t^l - \tilde{w}_t^h)}_{\text{relative wage}} \tag{17}$$

where $\psi_2 = \frac{1}{2} \sigma \lambda \frac{f^l}{2}$, $\psi_3 = \sigma \lambda \frac{f^l}{2} (1 - \frac{f^{ee}}{2})$ and variables without time subscript are the steady-state values.

The interest rate response is determined by fluctuations in wages and labor market transition rates. The former reflects the desire for consumption smoothing due to aversion to intertemporal substitution, while the latter represents the impact of the precautionary saving motive. Consumption-smoothing has a clear stabilizing effect on the economy. Following a negative productivity shock, workers seek to finance current consumption with future labor income as they expect it to grow in the next periods when the effect of the shock vanishes. Given workers' lack of insurance against income risk, the interest rate is further influenced by endogenous fluctuations in unemployment risk. With procyclical labor market tightness, adverse aggregate shocks depress job-finding, aggregate demand, and the real rate of interest. The second term in Equation (17) captures this destabilizing effect of unemployment risk, extensively discussed in theoretical HANK & SAM literature.

In contrast to the baseline HANK & SAM model, my equilibrium Equation (17) introduces precautionary terms associated with wage risk, which, as far as I am aware, has not been previously discussed in the literature. It demonstrates that the job ladder propagates aggregate shocks through two channels, namely the reallocation and relative wage channels.

[4] I interpret precautionary saving as any saving resulting from uncertainty about future income.

[5] I consider a steady state with positive vacancy posting, away from the liquidity trap. Ravn and Sterk (2021) demonstrate that, generally, HANK and SAM models can also have steady states associated with a liquidity trap and an unemployment trap.

Intuitively, the *reallocation channel* links savings to the rate of job-to-job transitions. When the job-to-job transition rate is low, the impact of a separation shock becomes pronounced as workers have a reduced probability of returning to a high-productivity state. Consequently, from the viewpoint of workers, a weak labor market entails the added risk of remaining in low-productivity matches. This risk prompts workers to increase precautionary savings further, which weakens aggregate demand and exerts a destabilizing influence on the economy. The *relative wage channel*, on the contrary, stabilizes the economy, as the difference between w_t^h and w_t^l tends to diminish, as discussed in Section 2. This reduction in wage disparities effectively mitigates the wage risk for high-productivity workers, leading to a decrease in precautionary savings.

The impact of the job ladder on aggregate demand remains ambiguous, as it is uncertain whether the reallocation or relative wage channels dominate or counterbalance each other. Moreover, the assumption of zero liquidity in this context may be misleading, as it overlooks the cyclical precautionary saving behavior of low-productivity workers. Notably, at the bottom of the job ladder, the influence of the relative wage channel undergoes a reversal, diminishing upward wage risk and potentially destabilizing the economy. Thus, to explore the impact of wage risk within a more realistic framework, I turn to quantitative analysis.

4.2 Quantitative results

In this subsection and throughout all subsequent sections, I depart from the assumption of two productivity states. Moreover, I relax the assumption of zero liquidity. In an economy with positive liquidity, employed workers across all productivity levels and unemployed workers can save in a unique liquid asset. The different histories of income risk realizations give rise to ex-post heterogeneity, thereby shaping the cross-section wealth distribution. The intensity of the precautionary saving motive becomes idiosyncratic, contingent upon individual wealth levels and labor earnings. I investigate the dynamic behavior of aggregates within this environment through a quantitative experiment. To this end, I perturb the economy with a zero-probability shock to aggregate productivity. Then, I allow productivity to converge deterministically toward its steady-state value.

Consider the economy remaining at a steady state before the shock occurs. At $t = 0$, all retailer firms experience an unexpected 1% decrease in productivity A_t. After the shock, productivity obeys the following law:

$$dA_t = \theta^A (A^{ss} - A_t) dt \tag{18}$$

where θ^A is a parameter of persistence. Figure 3 illustrates the economy's adjustment following the productivity shock in the baseline economy.

The initial adjustment of the economy is consistent with the mechanics of conventional HANK&SAM

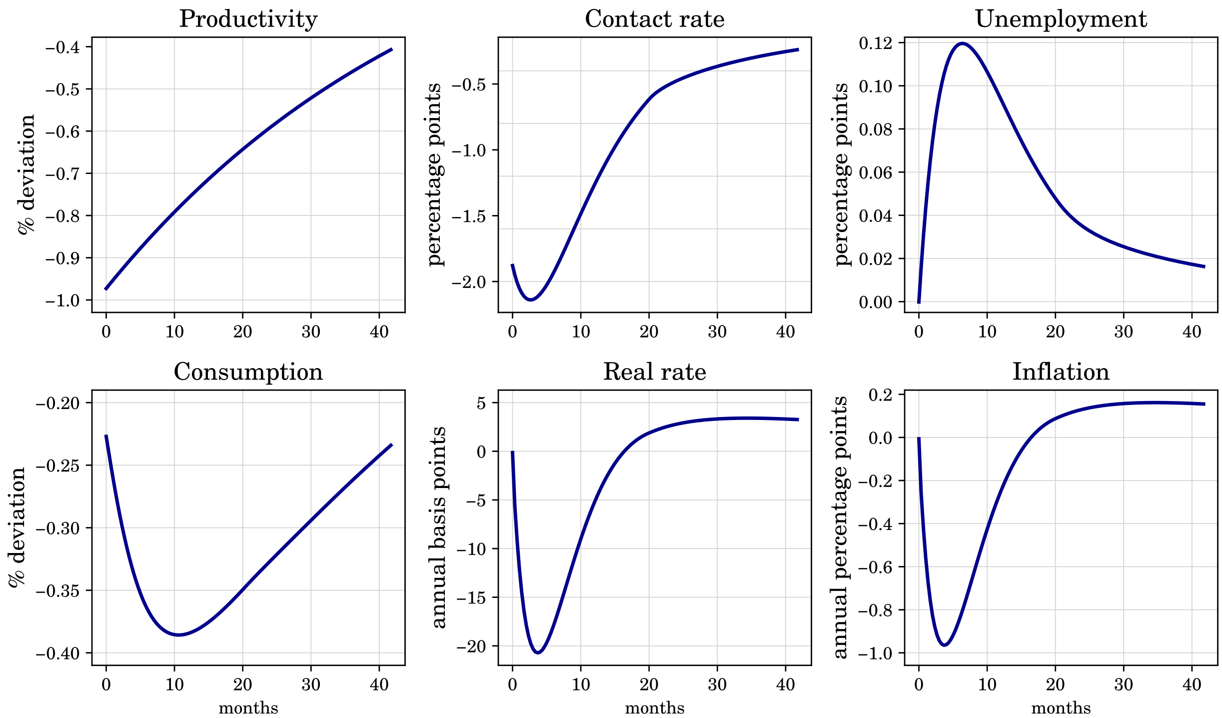


Figure 3: Response to a negative productivity shock in the baseline model

models extensively discussed in the literature. Following an adverse shock, a decline in productivity prompts retailers to decrease their demand for labor services, reducing the incentive for labor intermediaries to enter the market and post vacancies, which worsens aggregate labor market conditions. Low labor-market tightness has a dual effect on the dynamics of this economy, operating through (i) precautionary saving and (ii) compositional changes.

Initially, a weak labor market primarily impacts aggregate demand through the precautionary saving motive. As explained in Section 4.1, low contact rates increase both unemployment and wage risks, forcing risk-averse households to reduce their consumption. In a sticky-price economy, consumption cuts depress aggregate demand, prices, and interest rates, which in turn feed back into a weaker labor market. Thus, upon impact, the demand feedback stems mainly from the fear of unemployment and wage changes.

In contrast to models with zero liquidity, in economies with a non-degenerate wealth distribution, low labor market tightness influences aggregate demand through the composition effects as well. After one period, the economy's response is shaped not only by the income risk perceived by workers but also by actual compositional changes stemming from a weak labor market. As a result of this latter effect, approximately three quarters after the shock, the economy undergoes an overshoot before monotonically converging to its steady state.

4.3 Impact of wage risk

Next, I contrast the baseline model with two counterfactual scenarios: (i) where wages adjust proportionally, thus rendering the relative wage channel inoperative, and (ii) where the job-job transition rate remains constant over time, effectively shutting down both the relative wage and reallocation channels.

Following the approach of Kaplan, Moll and Violante (2018), I begin by expressing consumption as a function of a series of labor market transition rates, wages, and interest rates entering the worker problem:

$$C_t\left(\left\{f_t^{ee}, w_t, f_t, r_t\right\}_{t\geq 0}\right) = \int c_t\left(a_t, \frac{w_{x,t}}{w_{n,t}}, w_{n,t}, \left\{f_t^{ee}, \frac{w_{x,t}}{w_{n,t}}, w_{n,t}, f_t, r_t\right\}_{t\geq 0}\right) d\Phi_t(a,x) \tag{19}$$

The key step is to rewrite individual consumption as a function of the wage in the most productive state ($w_{n,t}$) and the ratio of the wage in the current productivity state to the wage in the most productive state ($\frac{w_{x,t}}{w_{n,t}}$). Next, I calculate the total differential of Equation (19) with respect to all variables that enter the worker problem, which allows me to decompose the consumption response at $t = 0$:

$$dC_0 = \int_0^\infty \frac{\partial C_t}{\partial f_t^{ee}} df_t^{ee} dt + \int_0^\infty \frac{\partial C_t}{\partial \frac{w_{x,t}}{w_{n,t}}} d\frac{w_{x,t}}{w_{n,t}} dt + \int_0^\infty \frac{\partial C_t}{\partial w_{n,t}} dw_{n,t} dt + \int_0^\infty \frac{\partial C_t}{\partial f_t} df_t dt + \int_0^\infty \frac{\partial C_t}{\partial r_t} dr_t dt \tag{20}$$

The first term represents the partial consumption response, dC_0, to changes in the contact rate f_t^{ee}; the second term corresponds to the partial consumption response to changes in the relative wages $\frac{w_{x,t}}{w_{n,t}}$; and the remaining three terms capture the consumption responses to changes in wages w_t, job-finding rates f_t, and interest rates r_t. By employing this decomposition, I quantify the contributions of the relative wage and reallocation channels to the aggregate consumption response, which correspond to the first two terms in Equation (20). To accomplish this, I construct a counterfactual equilibrium where I hold either the first or second terms in Equation (20) at their steady-state value while allowing the other terms to vary. I present the results in Figure 4.

Figure 4 demonstrates that the decline in consumption is less pronounced when wages adjust proportionally following the shock. Without the relative wage channel, households exhibit lower precautionary demand for assets, thereby dampening consumption response. In the second counterfactual scenario, where I assume a constant contact rate at its steady-state value, both the relative wage and reallocation channels are shut down. Consequently, the destabilizing effect of cyclical wage risk is mitigated, resulting in only moderate reductions in consumption. It's worth noting that the counterfactual path of the contact rate also influences dynamics through changes in the economy's composition, distorting aggregate savings due to the reduced share of workers in high- and middle-productivity matches. I elaborate on the role of the composition effect in the

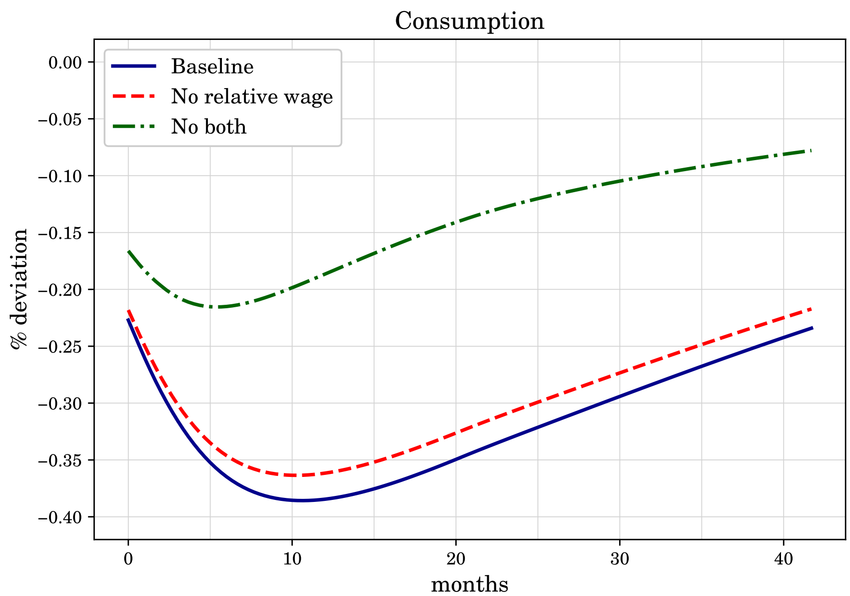


Figure 4: Impact of wage risk

following section.

Overall, the model demonstrates the quantitative significance of precautionary saving and asset accumulation resulting from cyclical wage risk. The relative wage and reallocation channels can amplify the unemployment and consumption response to productivity shocks by up to one-third compared to scenarios where cyclical wage risk is absent.

4.4 Impact of compositional changes

Similar to a zero-liquidity case, in my economy, *individual* consumption responses are determined by the relative strength of precautionary saving and consumption smoothing motives. However, with the assumption of positive liquidity, the *aggregate* consumption response is additionally influenced by compositional changes in the economy, alongside these two channels.

To understand the role of the composition effect in aggregate demand feedback, first consider a conventional HANK&SAM economy without the job ladder. Denoting saving policy by $\tilde{s}(.)$, the aggregate net savings supply in this economy can be expressed as a sum of savings of employed and unemployed workers: $\int \tilde{s}(a,e)da + \int \tilde{s}(a,u)da$. With the unemployment rate denoted as u, we can rewrite the aggregate savings as $(1-u)\frac{\int \tilde{s}(a,e)da}{1-u} + u\frac{\int \tilde{s}(a,u)da}{u}$, and ultimately as $(1-u)\bar{s}^e + u\bar{s}^u$, where $\bar{s}^e$ and $\bar{s}^e$ represent the mean savings of employed and unemployed workers, respectively.

Next, let's consider a shock leading to an increase in the unemployment rate, such as a negative productivity shock. We can derive a change in aggregate savings S^a in response to this shock:

$$\frac{\partial S^a}{\partial u} = \underbrace{(1-u)\frac{\partial \bar{s}^e}{\partial u}}_{\substack{\text{change in savings} \\ \text{employed}}} + \underbrace{u\frac{\partial \bar{s}^u}{\partial u}}_{\substack{\text{change in savings} \\ \text{unemployed}}} + \underbrace{\bar{s}^u - \bar{s}^e}_{\text{composition effect}} \tag{21}$$

The first term in Equation (21) encapsulates the adjustment in savings of employed workers in response to the shock. The direction of this adjustment hinges on whether the consumption smoothing or precautionary channel prevails, akin to the zero-liquidity case discussed in Section 4.1. The second term represents the savings response of the unemployed, governed by aversion to intertemporal substitution and aversion to risk, much like the adjustment observed in employed workers. This term appears in the economy with positive liquidity because unemployed workers are not necessarily borrowing-constrained.

The last two terms in Equation (21) account for the disparity in savings between unemployed and employed workers. This disparity arises because search and matching frictions affect the economy not only by changing perceived uncertainty but also by inducing compositional changes in the labor force. The negative productivity shock leads to an increase in the pool of unemployed, giving rise to the composition effect. In any realistic calibration, $\bar{s}^u \leq \bar{s}^e$, but the exact magnitude of the composition effect generally depends on the calibration and the tightness of the borrowing constraint in particular. In this context, a zero liquidity economy represents a limiting case where $\bar{s}^u = \bar{s}^e$ and compositional changes affect the economy solely through the dynamic path of labor market tightness. The recent paper by Cho (2023) discusses the quantitative implications of compositional changes for aggregate dynamics. In his estimated HANK and SAM model, the composition effect mutes down the precautionary demand feedback and contributes to excessive consumption volatility, an issue that does not arise in zero-liquidity economies.

The economy with a job ladder introduces an additional term in the composition effect. The change in aggregate savings now becomes:

$$\frac{\partial S^a}{\partial u} = \underbrace{\gamma^h(1-u)\frac{\partial \bar{s}^h}{\partial u}}_{\substack{\text{change in savings} \\ \text{high-employed}}} + \underbrace{\gamma^l(1-u)\frac{\partial \bar{s}^l}{\partial u}}_{\substack{\text{change in savings} \\ \text{low-employed}}} + \underbrace{u\frac{\partial \bar{s}^u}{\partial u}}_{\substack{\text{change in savings} \\ \text{unemployed}}} + \underbrace{\bar{s}^u - \gamma^h\bar{s}^h - \gamma^l\bar{s}^l}_{\text{composition effect}} \tag{22}$$

γ_l and γ_h represent the share of workers in low- and high-productivity states within the pool of the employed, while $\bar{s}^l$ and $\bar{s}^h$ denote the respective mean saving policies.

Next, I return to my quantitative model and examine the steady-state saving function across the three employment states: the saving function for unemployed households, the saving function for employed workers at the bottom of the job ladder (low-productivity matches), and the saving function for employed workers at the top of the job ladder (high-productivity matches). I depict all three saving functions in Figure 5.

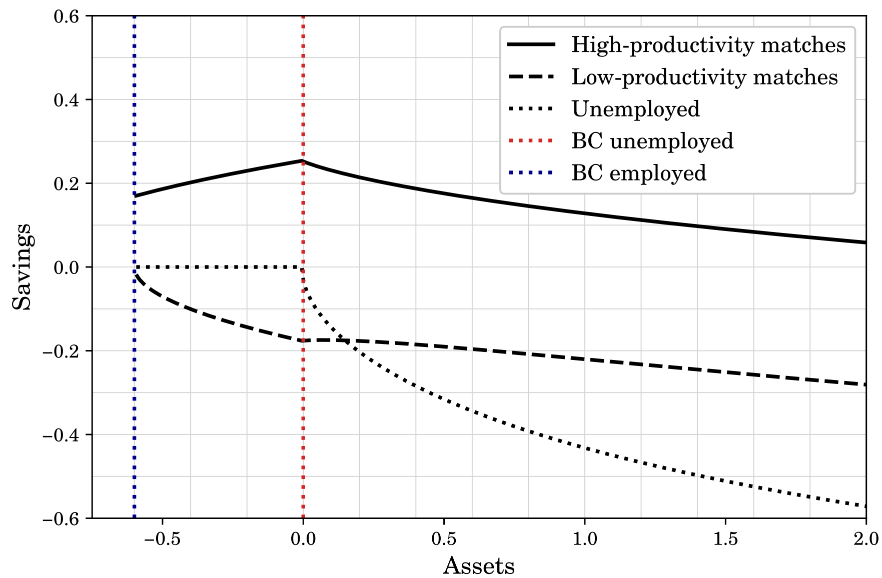


Figure 5: Policy function

While the savings of the employed in a high-productivity state always exceed the savings of the unemployed, expressed as $\bar{s}^u \leq \bar{s}^h$, this isn't necessarily the case for the low-productivity state. We observe that in the vicinity of the unemployed's borrowing limit, $\bar{s}^u > \bar{s}^l$. The reason behind this lies in the fact that the tightness of the borrowing constraint depends on the employment state of a worker, reflecting the ease of obtaining borrowing when a worker is more productive.
The sign of the composition channel becomes ambiguous in this environment. Eased borrowing constraints for low-productivity workers can potentially reverse the standard impact of compositional changes observed in a conventional HANK&SAM model without the on-the-job search. Intuitively, if the mass of unemployed workers with lower asset levels is significant, and new job matches predominantly have low productivity, the composition effect generates a countercyclical supply of savings. This, in turn, acts as a destabilizing force.
I conduct another counterfactual experiment to disentangle the dynamic effect of compositional changes in my model. To this end, I simulate the transition path of the economy following the shock by assuming a fixed distribution of workers across income and wealth. The results of this experiment are illustrated in Figure 6.
In a scenario with a fixed distribution, the economy converges to its steady state monotonically, contrasting with the baseline case. In the model where the composition effect is operative, the transition of workers from low-productivity jobs to unemployment generates an additional sup-

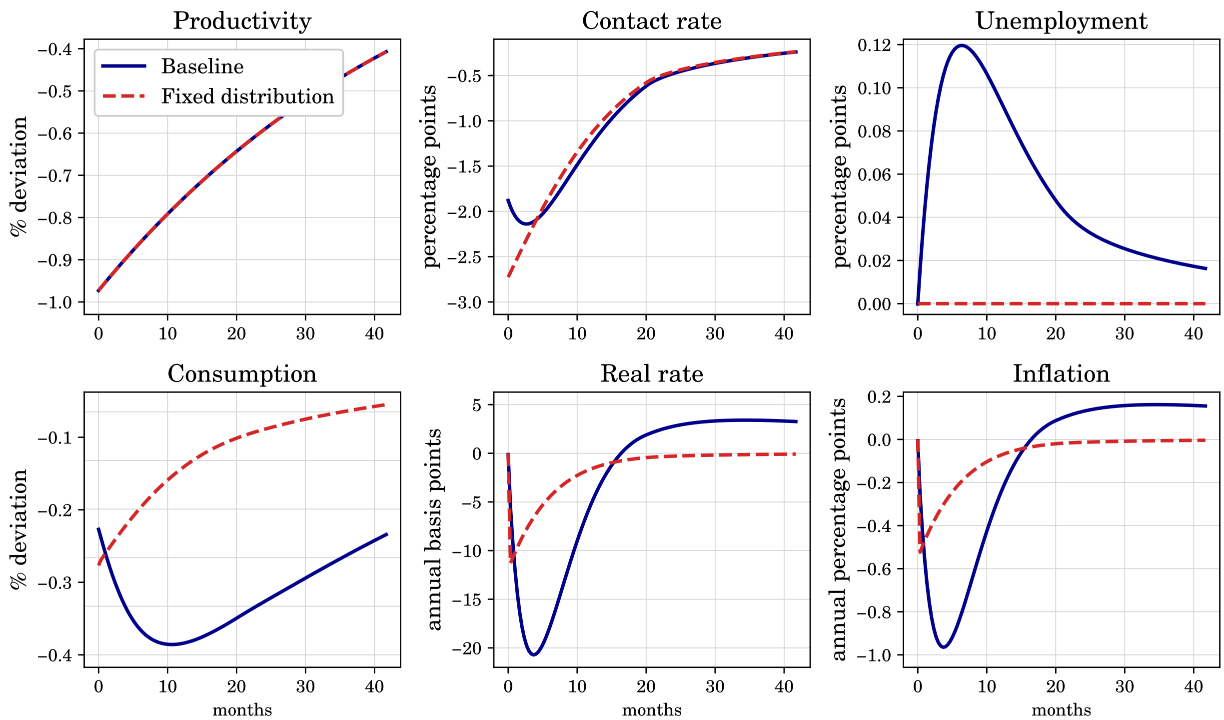


Figure 6: Effect of compositional changes

ply of savings. Over time, as the share of unemployed workers begins to converge back to a steady state, the change in composition triggers opposite dynamics, leading to an overshoot in the interest rate and inflation.

5 An Empirical Examination

The previous sections explore the role of wage risk stemming from the cyclical job ladder in propagating aggregate shocks. The preceding analysis demonstrates that the reallocation channel and the relative risk channel can enhance wage risk and have a destabilizing effect on the economy. Now, I take an empirical stance and look at these channels through the lens of the data. To this end, I compare key relationships between moments implied by my theoretical and quantitative models with those observed in the data. Importantly, this section doesn't aim to validate the model formally. Instead, it advocates for the plausibility of the channels' existence.

5.1 Real rate and on-the-job search

One indirect check of the income risk channel proposed by Ravn and Sterk (2021) involves examining the co-movement of labor market tightness and the real interest rate. The underlying idea is that if the precautionary channel prevails over the consumption smoothing motive, periods of a slack labor market should correspond to the low-interest rate episodes. Here, I adopt a similar approach by juxtaposing the real interest rate with the job-to-job transition rate Figure 7.

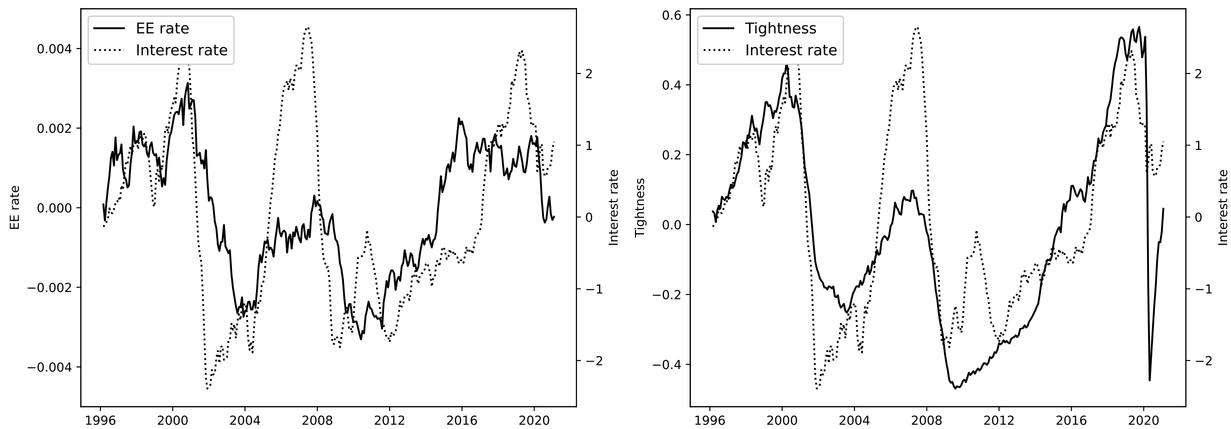


Figure 7: Real interest rate, job-to-job transition rate and labor market tightness

The left panel compares the real interest rate with the job-to-job transition rate. The left panel compares the real interest rate with the labor market tightness. The real interest rate is the Federal Funds rate minus CPI inflation.

Non-strikingly, the job-to-job transition rate co-moves with the real interest rate as the labor market tightness does so. What's surprising is the stronger observed correlation between the real interest rate and the job-to-job transition rate compared to its correlation with labor market tightness. The difference stems mainly from the period after the Great Recession when variations in the job-to-job transitions and labor market tightness rate were somewhat discontinued. Overall, this pattern indirectly suggests the additional impact of precautionary saving associated with cyclical labor reallocations on cyclical dynamics.

5.2 Relative wage channel

To look at the relative wage channel from the empirical standpoint, I adopt the approach outlined by Moscarini and Postel-Vinay (2017*b*). For this purpose, I derive a model-implied relationship between wage growth and confront it with the empirical counterpart.

First, I use Equation (1) to derive individual expected wage growth of workers between time t and $t + dt$.

$$E(\dot{w}_t(x)) = f^{ee} \int_x^{\bar{x}} \Big[w_t(s) - w_t \Big] dG(s) + (1 - f^{ee}) \dot{w}_t(x) \tag{23}$$

As Equation (23) demonstrates, a worker with productivity x experiences wage growth either due to job-to-job reallocation, which depends on the contact rate f^{ee}, or for reasons unrelated to reallocation. Next, I examine how the expected wage growth depends on the contact rate:

$$\frac{\partial E(\dot{w}_t(x))}{\partial f^{ee}} = \underbrace{\left[\int_x^{\bar{x}} \left[w_t(s) - w_t(x) \right] dG(s) - \dot{w}(x) \right]}_{>0} + \underbrace{\frac{\partial \dot{w}(x)}{\partial f^{ee}}}_{<0} \tag{24}$$

An increase in the contact rate has two opposing effects on the expected growth rate, stemming from different channels. Firstly, the impact of an increase in the contact rate is positive, as a higher contact rate implies a greater intensity of reallocation along the job ladder. This effect is captured by the first term in Equation (24). The second term in Equation (24) represents the poaching channel. As discussed in Section 2, for job stayers, an increase in the contact rate implies a decline in equilibrium wages, particularly at the bottom of the productivity distribution.

Next, I examine the theoretical relationship between wage growth and the contact rate as given by Equation (24). To this end, I estimate the correlation between wage growth and job-to-job transition rates in the data. In what follows, I closely follow the methodology of Postel-Vinay and Robin (2002) and estimate the following two-step specification.

Step 1. *Elimination of the composition effect*. My three variables of interest are wage growth (Δw_{it}), unemployment-employment (UE_{it}), and employment-employment (EE_{it}) transition. For each individual i, y_{it} denotes either the wage growth or the labor market transition indicator. I estimate the following equation:

$$y_{it} = X_{it}^{y} \beta_y + M_{it}^{t} \gamma_y + \varepsilon_{it} \tag{25}$$

where M is the interaction of dummies for age, race, gender, education, and time; X is a vector of individual characteristics of state of residence, occupation, industry, employer size, and union status. Then, I extract the market-time fixed effects:

$$\hat{\Phi}_{it}^{y} = M_{it}^{t} \hat{\gamma}_y \tag{26}$$

Step 2. *Estimating the correlation between the fixed effects*. Next, I regress the time-fixed effect of the wage growth on the time-fixed effects of labor market flows

$$\hat{\Phi}_{it}^{\Delta w} = \sum_{h \in \{UE, EE\}} \hat{\Phi}_{it}^{h} \delta_h + M_{it} \psi + \phi t + \eta_{it} \tag{27}$$

where t is the time dummy and η_{it} stands for the error term. The results of the estimation are reported in Table 4.

The first column in Table 4 indicates that the correlation between the job-to-job transition rate and wage growth surpasses the correlation with the job-finding rate. This result aligns with the findings of Faberman, Justiniano et al. (2015); Moscarini and Postel-Vinay (2017*b*); Karahan et al. (2017), who demonstrated that the employment-employment rate significantly outperforms the

Dependent variable: log change in monthly nominal earnings				
	(1)	(2)	(3)	(4)
EE rate	0.0424	.00806	-.01555	.00158
	(0.0006)	(.00026)	(.00043)	(.00091)
UE rate	-0.0005	-.00136	-.00132	-.00139
	(0.00004)	(.00002)	(.00002)	(.00002)
(EE rate) x age 36-45			.07746	
			(.00062)	
(EE rate) x age 46-60			.00309	
			(.00069)	
(EE rate) x age 60+			-.02368	
			(.00100)	
(EE rate) x educ2				-.02205
				(.00128)
(EE rate) x educ3				-.01516
				(.00121)
(EE rate) x educ4				-.00559
				(.00112)
(EE rate) x educ5				.03979
				(.00097)
	all	stayers	stayers	stayers

Table 4: Regression results

The table presents the result of estimating Equation (27). The first column presents the result for the whole sample, columns 2-4 only for job stayers. The data is from the Survey of Income and Program Participation covering 1996-2013 at a monthly frequency.

job-finding rate as a predictor variable for wage growth. Besides, these findings provide empirical support in favor of the strand of labor market theories originated from Burdett and Mortensen (1998). When the job-to-job transition rate is high, workers ascend the job ladder more rapidly and undergo a more substantial increase in wages. Consequently, wage growth is propelled by the cyclical dynamics of the job ladder rather than by cyclical fluctuations in the value of outside options, as observed in the framework proposed by Mortensen and Pissarides (1994).

The second column reveals a comparable correlation pattern between wage growth and labor market flows, observed not only for job-switchers but also for job-stayers. This finding is consistent with the view of Postel-Vinay and Robin (2002), suggesting that wage growth among job-stayers might be influenced by outside offers, thus prompting wage renegotiations.

Finally, in columns 3 and 4, I examine the same specification while controlling for age and education groups. The correlation between wage growth and the job-to-job transition rate is negative for both the youngest and oldest age groups, as well as for individuals with lower levels of education. I interpret this negative correlation through the lens of the poaching channel outlined in Section 2. As per Equation (24), it becomes apparent that the poaching channel should result in a negative correlation between wage growth and the job-to-job transition rate, particularly at the bottom of the job ladder. As this relation might be blurred by the renegotiation channel, we might still observe the negative correlation for specific groups of workers.

6 Conclusion

This paper explores the role of cyclical wage risk in transmitting aggregate shocks, adding a different perspective to the existing literature that predominantly focuses on unemployment risk. In my model, wage risk arises from cyclical reallocations across the job ladder. Within a zero-liquidity economy, I theoretically identify two channels through which the job ladder propagates the aggregate shocks: the reallocation and relative wage channels, which I argue are empirically plausible. Then, I demonstrate that both channels significantly impact cyclical dynamics even in more realistic setups. The quantitative model, which relaxes the assumption of zero liquidity, illustrates that the cyclical nature of wage risk enhances demand feedback, thereby amplifying employment and consumption responses by roughly one-third. This finding suggests that other aspects of the frictional labor market, beyond conventional unemployment risk, can also play an important role in propagating aggregate shocks when combined with imperfections in goods and financial markets.

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Precautionary saving, wage risk, and cyclical reallocation
Appendix

Mykhailo Matvieiev

A Analytical model

This appendix presents the derivations of the results from the analytical section. Specifically, I derive a wage equation for the model with endogenous destruction, which includes the model with exogenous destruction as a special case.

Each worker-firm match has an aggregate productivity z and an idiosyncratic productivity x. With Poisson intensity λ, an idiosyncratic shock (without memory) hits a match and changes x to some random value from distribution $dG(x)$. Firms have a reservation productivity R and destroy a job if $x < R$. From this setup, it is straightforward to move to a model with exogenous destruction by assuming that every idiosyncratic shock leads to the destruction of a match. Both models deliver the same wage equations.

Denoting U as the value of being unemployed and W as the value of being employed, the Hamiltonian-Jacobi-Bellman equations for the unemployed and employed workers are:

$$(r+\lambda)W(x) = w(x) + \lambda \int_R^{\bar{x}} W(s)dG(s) + \lambda G(R)U + \theta q(\theta)\left[\int_x^{\bar{x}} W(s)dG(s) - (1-G(x))W(x)\right]$$

$$rU = b + \theta q(\theta)\int_R^{\bar{x}} \left[W(s) - U\right] dG(s)$$

Denoting J and V as the values of firms with filled and vacant jobs, respectively, the Hamiltonian-Jacobi-Bellman (HJB) equations for these firms are:

$$rJ(x) = zx - w(x) + \lambda \int_R^{\bar{x}} J(s)dG(s) - ((1-G(x))\theta q(\theta) + \lambda)J(x)$$

$$rV = -zc + q(\theta)\left[\int_R^{\bar{x}} F(s)[J(s) - V]dG(s)\right]$$

Wage determined according to rule $\frac{W(x)-U}{J(x)} = \frac{\phi}{1-\phi}$, where ϕ is bargaining power or equivalently: $(1-\phi)W(x) - \phi J(x) = (1-\phi)U$. Consequently,

$$(r+\lambda+\theta q(\theta)(1-G(x)))(1-\phi)U =$$

$$(1-\phi)\left[w(x)+\lambda\int_R^{\bar{x}} W(s)dG(s)+\lambda G(R)U+\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)\right]-\phi\left[zx-w(x)+\lambda\int_R^{\bar{x}} J(s)dG(s)\right]=$$

$$w(x)-\phi zx+(1-\phi)\lambda G(R)U+\lambda\left[(1-\phi)\int_R^{\bar{x}} W(s)dG(s)-\phi\int_R^{\bar{x}} J(s)dG(s)\right]+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)=$$

$$w(x)-\phi zx+(1-\phi)\lambda G(R)U+\lambda\left[(1-\phi)\int_R^{\bar{x}} UdG(s)\right]+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)=$$

$$w(x)-\phi zx+(1-\phi)\lambda G(R)U+\lambda(1-\phi)U[G(\bar{x})-G(R)]+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)=$$

$$w(x)-\phi zx+\lambda(1-\phi)U+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)$$

$$(r+\lambda+\theta q(\theta)(1-G(x)))(1-\phi)U = w(x)-\phi zx+\lambda(1-\phi)U+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)$$

$$(r+\theta q(\theta)(1-G(x)))(1-\phi)U = w(x)-\phi zx+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)$$

$$r(1-\phi)U = w(x)-\phi zx+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} [W(s)-U]dG(s)$$

Now using the value for the unemployed:

$$(1-\phi)\left[b+\theta q(\theta)\int_R^{\bar{x}}\left[W(s)-U\right]dG(s)\right] = w(x)-\phi zx+(1-\phi)\theta q(\theta)\int_x^{\bar{x}} [W(s)-U]dG(s)$$

$$(1-\phi)b = w(x)-\phi zx-(1-\phi)\theta q(\theta)\int_R^{x} [W(s)-U]dG(s)$$

$$w(x) = (1-\phi)b+\phi zx+(1-\phi)\theta q(\theta)\int_R^{x} [W(s)-U]dG(s)$$

$$w(x) = (1-\phi)b+\phi zx+\phi\theta q(\theta)\int_R^{x} J(s)dG(s)$$

$$w(x) = (1-\phi)b+\phi zx+\phi\theta q(\theta)\int_R^{\bar{x}} J(s)dG(s)-\phi\theta q(\theta)\int_x^{\bar{x}} J(s)dG(s)$$

now using the FEC obtain:

$$w(x) = (1-\phi)b+\phi z(x+\theta cF(\xi)^{-1})-\underbrace{\phi\theta q(\theta)\int_x^{\bar{x}} J(s)dG(s)}_{\Delta}$$

Next, the goal is to find Δ. Taking HJB of employed workers and dividing both parts by $\theta q(\theta)$:

$$\frac{r+\lambda+(1-G(x))\theta q(\theta)}{\theta q(\theta)}W(x) = \frac{1}{\theta q(\theta)}\left[w(x)+\lambda\int_R^{\bar{x}}\left[W(s)-U\right]dG(s)+\lambda U\right]+\int_x^{\bar{x}} W(s)dG(s)$$

Denoting:

$C(x) = \frac{r+\lambda+(1-G(x))\theta q(\theta)}{\theta q(\theta)}$, which depends on x through $G(x)$

$A(x) = \frac{1}{\theta q(\theta)}\left[w(x)+\lambda\int_R^{\bar{x}}\left[W(s)-U\right]dG(s)+\lambda U\right]$, which depends on x through $w^s(x)$

Rewrite:

$$C(x)W(x) = A(x) + \int_x^{\bar{x}} W(s)dG(s)$$

$$C'(x)W(x) + C(x)W'(x) = A(x) - W(x)G'(x)$$

$$-G'(x)W(x) + C(x)W'(x) = A'(x) - W(x)G'(x)$$

$$C(x)W'(x) = \frac{w'(x)}{\theta q(\theta)}$$

Maximization of Nash product implies:

$$W'(x) = \frac{\phi}{(1-\phi)}J'(x)$$

From the previous wage equation

$$w'(x) = \phi p + \phi\theta q(\theta)J(x)G'(x)$$

Plug in $W'(x)$ and $w'(x)$

$$C(x)\frac{\phi}{(1-\phi)}J'(x) = \frac{\phi}{\theta q(\theta)}p + \phi J(x)G'(x)$$
$$C(x)\frac{\phi}{(1-\phi)}J'(x) = \frac{\phi}{\theta q(\theta)}p - \phi J(x)C'(x)$$
$$C(x)J'(x) + (1-\phi)J(x)C'(x) = \frac{(1-\phi)}{\theta q(\theta)}p$$

The RHS does not depend on x. First, I solve for the case where RHS = 0

$$\frac{J'(x)}{J(x)} = -(1-\phi)\frac{C'(x)}{C(x)}$$

$$J(x) = \Gamma C(x)^{-(1-\phi)}$$

Using the method of variation

$$J'(x) = \Gamma' C(x)^{-(1-\phi)} - (1-\phi)\Gamma C(x)^{-(1-\phi-1)}C'(x)$$

$$C(x)J'(x) = \Gamma' C(x)^{-(1-\phi)}C(x) - (1-\phi)\Gamma C(x)^{-(1-\phi)}C'(x)$$

Plug in the equation

$$\Gamma' C(x)^{-(1-\phi)}C(x) - (1-\phi)\Gamma C(x)^{-(1-\phi)}C'(x) + (1-\phi)J(x)C'(x) = \frac{(1-\phi)}{\theta q(\theta)}z$$
$$\Gamma' C(x)^{-(1-\phi)}C(x) - (1-\phi)\Gamma C(x)^{-(1-\phi)}C'(x) + (1-\phi)J(x)C'(x) = \frac{(1-\phi)}{\theta q(\theta)}z$$
$$\Gamma' C(x)^{\phi} = \frac{(1-\phi)}{\theta q(\theta)}z$$
$$\Gamma = \frac{(1-\phi)}{\theta q(\theta)}z\int_R^x \frac{1}{C(x)^{\phi}}dx$$

To repeat, $\Delta = \phi\theta q(\theta)\int_x^{\bar{x}} JdG(s)$
To solve for Δ, an exact functional form for G is needed. Assuming uniform distribution U[0,$\bar{x}$].

$$C(x) = \frac{r+\lambda}{\theta q(\theta)} + 1 - G(x) = \frac{r+\lambda}{\theta q(\theta)} + 1 - \frac{x}{\bar{x}}$$

$$dC(x) = -\frac{1}{\bar{x}}dx$$

$$\int_R^x \frac{1}{C^\phi}dx = \bar{x}\int_R^x \frac{1}{C^\phi}dC = \bar{x}\frac{1}{\phi-1}\frac{1}{C^{\phi-1}}\Big|_{C(x)}^{C(R)} = \bar{x}\frac{1}{\phi-1}\big(-C(x)^{1-\phi} + C(R)^{1-\phi}\big)$$

$$J = \frac{\bar{x}p}{\theta q(\theta)}\left[-1 + \left[\frac{C(R)}{C(x)}\right]^{1-\phi}\right]$$

$$\Delta = \phi\bar{x}p\int_{\bar{x}}^x \left[-1 + \left[\frac{C(R)}{C(x)}\right]^{1-\phi}\right]dG$$

$$\frac{\Delta}{\phi\bar{x}z} = \int_{\bar{x}}^x \left[1 - \left[\frac{C(R)}{C(x)}\right]^{1-\phi}\right]dC = 1 - \frac{x}{\bar{x}} - C(R)^{1-\phi}\int_{\bar{x}}^x C(x)^{-(1-\phi)}dC =$$

$$= 1 - \frac{x}{\bar{x}} - C(R)^{1-\phi}\frac{1}{\phi}\Big(C(\bar{x})^\phi - C(x)^\phi\Big)$$

$$\Delta = \phi z(\bar{x} - x) + \bar{x}zC(R)\left[\frac{C(x)^\phi - C(\bar{x})^\phi}{C(\bar{R})^\phi}\right]$$

B Zero-liquidity economy

In the limiting case of a zero-liquidity economy, let's consider a scenario where events are timed differently within a period. Initially, at the start of the period, workers encounter the risk of separation with the probability λ. The remaining period is divided into two subperiods. During each subperiod, unemployed workers have a chance of finding a job in either a low-productivity match with probability $f_t^l\frac{1}{2}$, or a high-productivity match with probability $f_t^h\frac{1}{2}$. For workers in low-productivity matches, there exists a probability, $f^{ee}\frac{dt}{2}$, of transitioning to a more productive job within a subperiod.
In this timing convention, for a worker in the high-productivity state at the beginning of time t, the probability of transitioning to the unemployment state at the beginning of time $t+1$ is $\lambda(1 - f_t^h - f_t^l)$. The corresponding probability of transitioning to the low-productivity state at the beginning of time $t+1$ is $\lambda\frac{f_t^l}{2}((1 - \frac{f_t^{ee}}{2}) + (1 - \frac{f_t}{2}))$. Defining $f_t = f_t^h + f_t^l$ yields the probabilities from Equation (16).